THE AMERICAN MATHEMATICAL MONTHLY.

Entered at the Post-office at Springfield, Missouri, as Second-class Mail Matter.

VOL. IV. AUGUST-SEPTEMBER, 1897. NOS. 8-9.

BIOGRAPHY.

PROFESSOR DE VOLSON WOOD.

"PROFESSOR WOOD was a man of wide and enviable reputation. It had been the fortune of many generations of students to sit under his teachings, he had written books which are standard in the technical schools and among engineers, and he had been active all his life in written and spoken discussions before the several societies of which he was a member. Furthermore, he had personal qualities which impressed themselves promptly and strongly upon those who came in contact with him, and as a consequence of all these conditions he was one of the best-known professors in the United States. But beyond all that lay extraordinary ability as a mathematician and as an analyst, remarkable strength and simplicity of character, and a genius for teaching which made his reputation a good deal more than temporary or local.

Professor Wood was a man of considerable practical mechanical ability, but that ability had never been turned to very important results. His powers as a mathematician, however, have given him a permanent place in the literature of engineering, and no student of the higher mathematics of engineering can remain ignorant of the name of DeVolson Wood. But his real greatness was as a teacher. In one sense perhaps that is a misfortune for a man, because he leaves no monument except in the hearts and the minds of the men who actually came under his personal influence. His fame becomes a tradition, fading away and gradually disappearing. On the other hand, is this not the very best work that a man can do in the world—the work of a really strong and sound teacher?

It would be difficult to sum up in a few words all the qualities which made Professor Wood great as a teacher, but the fundamental quality was his own downright sincerity and his faith in his own work; his mind knew only one test, and that was the truth. To him things were either right or they were wrong, and facts were facts or they were not facts, and he saw no occasion for trying to find any middle ground. But the pursuit of the truth is often enough an arid enterprise, and a man needs more than his own sincerity to get young men to follow him eagerly in that enterprise; and Professor Wood did get his students to work with alacrity, with eagerness, with enthusiasm. A strong element in this was his own rugged and wholesome enthusiasm; another was his air. His solid and robust figure, his keen eye and square jaw, his frank and ready smile—all these were part of his influence on the young men. Added to the genuineness which appeared in all his speech and all his manner was a gift of geniality. The youth who came in contact with him could not help feeling that he stood before a real man, a man strong and sound, mentally and physically; and while youth is not very analytical it is impressed by a man of such quality without knowing why it is impressed. The writer of these words, who had the fortune to sit under Professor Wood four years in civil engineering, can testify that no other teacher ever gave him such hard lessons or ever got out of him so good recitations, and yet there was no sense of hardship in it. It seemed a natural and inevitable thing to work about five times as hard for Professor Wood as for any other teacher, and this perhaps was largely a result of his own enthusiasm in the work. He had furthermore a gift of personal interest in his students. Probably a very small percentage of his pupils—and they must have been unworthy students at that—failed to feel that Professor Wood had a particular personal interest in them. It was not that he took any special trouble with any one man, but he was always able to carry a man's personality in his mind, and he seemed always to be interested in knowing something about a man's career, And so it came about that his influence on the lives of his students did not cease when they left his class-room.

Professor Wood was an active and sincere Christian gentleman, always interested in good work and always exerting a good influence in the community about him. Among a select body of students his name will be known and honored for generations to come as the name of a clear and able writer on the mathematics and mechanics of engineering; among a great body of teachers, students, engineers, and administrators he is remembered in gratitude and love as a strong and wholesome and stimulating friend." *From the Railroad Gazette of July* 2, 1897.

Professor Wood was born near Smyrna, New York, on June 1, 1832, and died at Hoboken, New Jersey, June 27, 1897. He began teaching in 1849, teaching for three terms in Smyrna. In 1853, he graduated from the Albany State Normal School. During the same and the following year he was principal at Napanoch. He was assistant professor of mathematics in Albany Normal, 1854-5, assistant instructor at the Renselaer Polytechnic Institute, Troy, 1855-7,

from which he received the degree of Civil Engineer. Hamilton College conferred the degree of Master of Arts in 1859.

At the University of Michigan he was professor from 1857 to 1872, receiving the degree of Master of Science in the second year of his professorship. Through his labors the department of civil engineering was organized. He became professor of mathematics and mechanics at Stevens Institute of Technology, Hoboken, New Jersey, in 1872, and upon the withdrawal of Prof. R. H. Thurston, to become president of Sibley College, Cornell, he became professor of mechanical engineering, which position he was holding at the time of his death.

He was a member of the American Society of Civil Engineers from 1871 to 1885, also of American Association for the Advancement of Science, since 1879, and its vice president in 1885. He was a member of the American Mathematical Society, and of the Society of Mechanical Engineers, and an honorary member of the Society of Architects. He was the first president of the Society for the Promotion of Engineering Education, started in Chicago at the time of the World's Fair.

He was engineer of the ore-dock, Marquette, Michigan, in 1864, and inventor of a steam rock drill and air compressor.

He contributed articles to the New York Teacher, Johnson's Cyclopædia, Appleton's Cyclopædia of Mechanics, the London Philosophical Magazine, Van-Nostrand's, The American Engineer, Michigan Journal of Education, Journal of Franklin Institute, Railroad Gazette, of which his son is now one of the editors; The Mining and Engineering Journal, Science, The Mathematical Visitor, The Analyst, The Annals of Mathematics, THE AMERICAN MATHEMATICAL MONTHLY, and other magazines.

He was the author of Trusses, Bridges and Roofs, published in 1872, Wood's Edition of Mahan's Civil Engineering, Treatise on the Resistance of Materials, Elements of Analytical Mechanics, Wood's Edition of Magnus' Lessons in Elementary Mechanics, Coördinate Geometry and Quaternions, Key and Supplement to Elements of Mechanics, and to the Mechanics of Fluids, Trigonometry, Turbines, and in 1887 he published one of the greatest of his books, Thermodynamics, which has entered a number of universities and gone through several editions.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from June-July Number.]

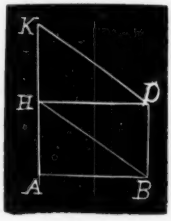
SCHOLION I. And this it is, that I said before in Cor. II. after XXV of this; obviously that no place would remain over for the hypothesis of acute angle, or Euclidean Geometry would be most exactly established, if any two straights existing in the same plane, as suppose AX, BX, which the straight AB meeting (the point B being assumed at a distance from the point A as great as you choose) makes with them toward the same parts of the points X two angles less than two right angles, if (I say) nowhere at another place (this standing) they can admit a common perpendicular.

For then these two AX, BX mutually approach each other ever more, indeed either within a certain determinate limit, as in XXV of this, or without any certain limit, and therefore even to meeting, anyhow after infinite production, as in this XXVII.

But it holds that in either of the aforesaid cases the destruction of the hypothesis of acute angle has now been shown. Quod intendebatur.

SCHOLION II. And again this it is, that I promised at the end of Scholium IV after XXI of this, as from the very terms clearly shines out.

SCHOLION III. Moreover I could wish here to be observed the difference between this proposition and the preceding XVII. For there (recall Fig. 15) has been shown the destruction of the hypothesis of acute angle, if (the straight AB being as small as you choose) every BD erected at whatever acute angle, must at length meet in some point K the perpendicular AH produced.



But here (viceversâ) in fact is permitted the designation of however most small an acute angle at the point A, while still the sect AB to which is to be erected the indefinite perpendicular BX, may be taken of any length whatever.

[To be Continued.]

ON THE COMPLEX ROOTS OF NUMERICAL EQUATIONS OF THE THIRD AND FOURTH DEGREE.

By A. C. BURNHAM, Berlin, Germany.

The real roots of a numerical equation can, as is well known, be found to any desired degree of accuracy by Horner's method of approximation. The complex roots as well can, for cubic and biquadratic equations, be very easily found by the same method. In fact a single application of Horner's method is in these cases sufficient for determining all the roots to any desired number of decimal places, whether the roots be positive or negative, commensurable or incommensurable, real or complex.

THE CUBIC EQUATION.

Let the cubic equation

$$x^3+a_1x^2+a_2x+a_3=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (A),$$

have the roots c, $a+bi$, $a-bi$, since one root must be real, where $i=\sqrt{-1}$ and a, b, c are real. The sums of the products of the roots one, two, and three at a time are equal respectively to $-a_1$, a_2, $-a_3$. That is

$$a+bi+a-bi+c=-a_1,$$

$$(a+bi)(a-bi)+(a+bi)c+(a-bi)c=a_2,$$

$$(a+bi)(a-bi)c=-a_3,$$

or

$$2a+c=-a_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

$$a^2+b^2+2ac=a_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2).$$

$$(a^2+b^2)c=-a_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3).$$

From these three equations it is not difficult to get the following:

$$8a^3+8a_1a^2+2(a_1^2+a_2)a+a_1a_2-a_3=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{I.}$$

$$c=-(a_1+2a) \text{ or } a=-\tfrac{1}{2}(c+a_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{II.}$$

$$\pm\sqrt{\frac{a_3-a_1a^2-2a^3}{2a+a_1}}=b=\pm\sqrt{\frac{-a_3}{c}-a^2}=\pm\sqrt{\frac{-a_3-\frac{1}{4}c(c+a_1)^2}{c}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{III.}$$

Equations I, II, III give all the roots to any desired degree of accuracy. One may find c from the given equation (A) by Horner's method, or a from

equation I by the same method, according to which is the easier. The a or c and b are given by II and III by a mere substitution. It is, of course, immaterial whether the positive or negative value of b be taken, since, in any case, both are used. b will be imaginary only when the original equation (A) has all three roots real. It is also of no consequence which of the three values for a given by equation I be taken, but I will in no case have a greater number of real roots than the given equation (A).

EXAMPLE. Find the roots of $x^3-2x-5=0$.

The one real root c, easily found by Horner's method is, $c=2.0945+$.

We have moreover, $a_1=0$, $a_2=-2$, $a_3=-5$. Therefore, $a=-\frac{1}{2}(c+a_1)$ $=-1.0472+$, and $b=\sqrt{\frac{5}{2.0945}-(-1.0472)^2}=1.123$.

The roots therefore are $-1.0472\pm1.123\sqrt{-1}$ and 2.0945.

In this example, equation I. takes the form

$$8a^3-4a+5=0,$$

which has the one real root $a=1.0472+$. This is the same result as above.

THE BIQUADRATIC EQUATION.

Let the roots of the biquadratic equation

$$x^4+a_1x^3+a_2x^2+a_3x+a_4=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (B).$$

be $a\pm bi$, $c\pm di$. We then have as before

$$a+bi+a-bi+c+di+c-di=-a_1,$$

$$(a+bi)(a-bi)+(a+bi)(c+di)+(a+bi)(c-di)+(a-bi)(c+di)$$
$$+(a-bi)(c-di)+(c+di)(c-di)=a_2,$$

$$(a+bi)(a-bi)(c+di)+(a+bi)(a-bi)(c-di)+(a+bi)(c+di)(c-di)$$
$$+(a-bi)(c+di)(c-di)=-a_3,$$

$$(a+bi)(a-bi)(c+di)(c-di)=a_4,$$

or
$$2(a+c)=-a_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

$$(a^2+b^2)+(c^2+d^2)+4ac=a_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

$$2c(a^2+b^2)+2a(c^2+d^2)=-a_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3).$$

$$(a^2+b^2)(c^2+d^2=a_4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4).$$

From these four equations we find

$$a=-\tfrac{1}{2}(2c+a_1) \quad \ldots\ldots (5).$$

$$c=-\tfrac{1}{2}(2a+a_1) \quad \ldots\ldots (6).$$

$$c^2+d^2=u=\frac{2ca_2+a_3+4c^2(2c+a_1)}{4c+a_1} \quad \ldots\ldots \text{I.}$$

$$a^2+b^2=t=-\frac{2au+a_3}{2c}=\frac{a_4}{u} \quad \ldots\ldots \text{II.}$$

If we now eliminate u and a by means of 5, I and II, we have the following equation of the sixth degree for c:

$$64c^6+96a_1c^5+16(3a_1^2+2a_2)c^4+8(4a_1a_2+a_1^3)c^3$$
$$+4(a_2^2+2a_1^2a_2+a_1a_3-4a_4)c^2+2(a_1a_2^2+a_1^2a_3-4a_1a_4)c$$
$$+a_1a_2a_3-a_1^2a_4-a_3^2=0 \quad \ldots\ldots \text{III.}$$

From I and II we have moreover,

$$d=\pm_1\sqrt{u-c^2} \quad \ldots\ldots (7).$$

$$b=\pm_1\sqrt{t-a^2} \quad \ldots\ldots (8).$$

Therefore, after getting a single value of c from III by Horner's method, a, u, t, d, and b follow respectively from (5), I, II, (7), and (8) by mere substitutions, and thus a single application of Horner's method suffices to find all of the roots, no matter what their character.

If the equation (B) has no real roots, then III will have only two real roots. They are separately the values for a and c, and either can be taken for c. That is, the equation of the sixth degree giving a is the same as III giving c.

EXAMPLE. Find the roots of the equation

$$x^4-6x^3+18x^2-30x+25=0.$$

In this equation $a_1=-6$, $a_2=18$, $a_3=-30$, $a_4=25$.
We have therefore as equation III,

$$4c^6-36c^5+144c^4-324c^3+425c^2-303c+90=0.$$

It is immediately seen that one is a root of this equation, therefore $c=1$, from which there follows,

from (5), $a=-\tfrac{1}{2}(2-6)=2$,

from I, $u=[2.18-30+4(2-6)]/[-2]=5$,

from II, $t=a_4/u=\frac{25}{5}=5$,

from (7), $d=\pm_1\sqrt{5-1}=\pm 2$,

from (8), $b=\pm_1\sqrt{5-4}=\pm 1$.

The roots are therefore $2\pm i$, $1\pm 2i$.

The value 2 for a satisfies the equation III as it should, and 1 and 2 are the only real roots which III possesses.

If the given equation (B) has two real roots and both are known to any desired degree of accuracy, the two other roots are very easily found. Put

$$a+bi=h$$
$$a-bi=k$$

where h and k are known. Then

$$a=\tfrac{1}{2}(h+k),$$
$$b=-\tfrac{1}{2}(h-k)i,$$
$$c=-\tfrac{1}{2}(h+k+a_1) \text{ from (5)},$$

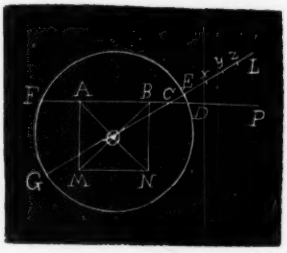
and u is found from I and d from (7) as before. Thus the roots are all determined.

If all of the roots of (B) are real, they will be equally well given by the first method above. In this case b and d will be imaginary.

A DEVICE FOR EXTRACTING THE SQUARE ROOT OF CERTAIN SURD QUANTITIES.

By ROBERT J. ALEY, A. M., Ph. D., Professor of Mathematics, University of Indiana, Bloomington, Indiana.

$ABMN$ is a square. OL is an arm revolving freely about O. This arm beyond C is divided into equal parts at E, x, y, z, etc.



To determine the character of the divisions made on FP by the points of division on OL as OL revolves. Call the side of the square AB, $2a$; BC, b; CE, c; and CD, x.

Then $OC=_1\sqrt{a^2+(a+b)^2}$.

$$GC=2_1\sqrt{a^2+(a+b)^2}+c.$$

$FC=2(a+b)+x.$

From the properties of two intersecting chords we have,

$$x\{x+2(a+b)\}=c\{c+2\sqrt{a^2+(a+b)^2}\}$$

$$x^2+2(a+c)x+(a+b)^2=a^2+2ab+b^2+c^2+2c\sqrt{a^2+(a+b)^2}$$

$$x+(a+b)=\sqrt{(a+b)^2+c^2+2c\sqrt{a^2+(a+b)^2}}.$$

Suppose that we examine the results when integral values are given to the constants.

Put $a=c=1$, $b=0$. (Let c take successively the values 1, 2, 3, 4, ... etc.)

Then $x+1=\sqrt{2+2\sqrt{2}}$,

$x+1=\sqrt{5+4\sqrt{2}}$,

$x+1=\sqrt{10+6\sqrt{2}}$,

$x+1=\sqrt{17+8\sqrt{2}}$, etc.,

and the law of the series is readily seen.

Put $a=c=b=1$, and let c vary as before.

$x+2=\sqrt{5+2\sqrt{5}}$,

$x+2=\sqrt{8+4\sqrt{5}}$,

$x+2=\sqrt{13+6\sqrt{5}}$,

$x+2=\sqrt{20+8\sqrt{5}}$, etc.

The law is again evident.

Put $a=1$, $b=2$, and let c vary.

$x+3=\sqrt{10+2\sqrt{10}}$,

$x+3=\sqrt{13+4\sqrt{10}}$,

$x+3=\sqrt{18+6\sqrt{10}}$,

$x+3=\sqrt{25+8\sqrt{10}}$, etc.

The law is again evident.

Put $a=1$, $b=3$, and let c vary.

from II, $t=a_4/u=\frac{25}{5}=5$,

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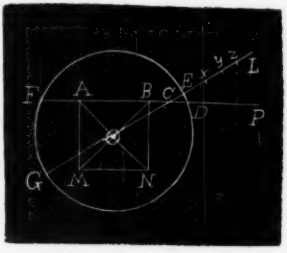

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Put $a=c=b=1$, and let c vary as before.

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The law is again evident.

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$$x+3=\sqrt{25+8\sqrt{10}}, \text{ etc.}$$

The law is again evident.

Put $a=1$, $b=3$, and let c vary.

$$x+4=\sqrt{17+2\sqrt{17}},$$

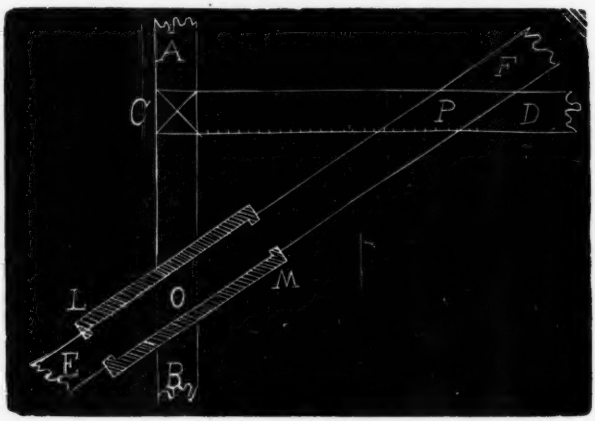
$$x+4=\sqrt{20+4\sqrt{17}},$$

$$x+4=\sqrt{25+6\sqrt{17}}.$$

These examples show how the various series will be found.

From the previous considerations we at once have the data for the construction of a simple mechanical device for the extraction of roots of certain surd quantities.

AB is an upright so arranged that *CD* will slide up and down always parallel to itself. It is accurately marked to scale so that *CD* may be set at any de-



sired a. *FE* works in a slide *LM* which is free to rotate about *O*. It is accurately ruled to scale from *P* to *F*. By sliding it in *LM*, *P* may be set at any desired $a+c$. *CD* is ruled to scale and is also provided with a diagonal scale, so that by the use of dividers, results may be read to hundredths. When the instrument is set at any chosen a and b, all the roots for that set may be read off at once.

Tables may be easily constructed. A few samples are here given.

The a's are read in the vertical columns, the b's horizontally, and in the squares the c's take successively the values 1, 2, 3, etc. But three terms are given in each square, enough to make the law perfectly evident.

TABLE FOR INTEGRAL VALUES OF a, b, AND c.

a =	b = 0	1	2	3	4	5	6	7	8
1	$\sqrt{2+2\sqrt{2}}$ $\sqrt{5+4\sqrt{2}}$ $\sqrt{10+6\sqrt{2}}$	$\sqrt{5+2\sqrt{5}}$ $\sqrt{8+4\sqrt{5}}$ $\sqrt{13+6\sqrt{5}}$	$\sqrt{10+2\sqrt{10}}$ $\sqrt{13+4\sqrt{10}}$ $\sqrt{18+6\sqrt{10}}$	$\sqrt{17+2\sqrt{17}}$ $\sqrt{20+4\sqrt{17}}$ $\sqrt{25+6\sqrt{17}}$	$\sqrt{26+2\sqrt{26}}$ $\sqrt{29+4\sqrt{26}}$ $\sqrt{34+6\sqrt{26}}$	$\sqrt{37+2\sqrt{37}}$ $\sqrt{40+4\sqrt{37}}$ $\sqrt{45+6\sqrt{37}}$	$\sqrt{50+2\sqrt{50}}$ $\sqrt{53+4\sqrt{50}}$ $\sqrt{58+6\sqrt{50}}$	$\sqrt{65+2\sqrt{65}}$ $\sqrt{68+4\sqrt{65}}$ $\sqrt{73+6\sqrt{65}}$	$\sqrt{82+2\sqrt{82}}$ $\sqrt{85+4\sqrt{82}}$ $\sqrt{90+6\sqrt{82}}$
2	$\sqrt{5+2\sqrt{8}}$ $\sqrt{8+4\sqrt{8}}$ $\sqrt{13+6\sqrt{8}}$	$\sqrt{10+2\sqrt{13}}$ $\sqrt{13+4\sqrt{13}}$ $\sqrt{18+6\sqrt{13}}$	$\sqrt{17+2\sqrt{20}}$ $\sqrt{20+4\sqrt{20}}$ $\sqrt{25+6\sqrt{20}}$	$\sqrt{26+2\sqrt{29}}$ $\sqrt{29+4\sqrt{29}}$ $\sqrt{34+6\sqrt{29}}$	$\sqrt{37+2\sqrt{40}}$ $\sqrt{40+4\sqrt{40}}$ $\sqrt{45+6\sqrt{40}}$	$\sqrt{50+2\sqrt{53}}$ $\sqrt{53+4\sqrt{53}}$ $\sqrt{58+6\sqrt{53}}$	$\sqrt{65+2\sqrt{68}}$ $\sqrt{68+4\sqrt{68}}$ $\sqrt{73+6\sqrt{68}}$	$\sqrt{82+2\sqrt{85}}$ $\sqrt{85+4\sqrt{85}}$ $\sqrt{90+6\sqrt{85}}$	
3	$\sqrt{10+2\sqrt{18}}$ $\sqrt{13+4\sqrt{18}}$ $\sqrt{18+6\sqrt{18}}$	$\sqrt{17+2\sqrt{25}}$ $\sqrt{20+4\sqrt{25}}$ $\sqrt{25+6\sqrt{25}}$	$\sqrt{26+2\sqrt{34}}$ $\sqrt{29+4\sqrt{34}}$ $\sqrt{34+6\sqrt{34}}$	$\sqrt{37+2\sqrt{45}}$ $\sqrt{40+4\sqrt{45}}$ $\sqrt{45+6\sqrt{45}}$	$\sqrt{50+2\sqrt{58}}$ $\sqrt{53+4\sqrt{58}}$ $\sqrt{58+6\sqrt{58}}$	$\sqrt{65+2\sqrt{73}}$ $\sqrt{68+4\sqrt{73}}$ $\sqrt{73+6\sqrt{73}}$	$\sqrt{82+2\sqrt{90}}$ $\sqrt{85+4\sqrt{90}}$ $\sqrt{90+6\sqrt{90}}$		
4	$\sqrt{17+2\sqrt{32}}$ $\sqrt{20+4\sqrt{32}}$ $\sqrt{25+6\sqrt{32}}$	$\sqrt{26+2\sqrt{41}}$ $\sqrt{29+4\sqrt{41}}$ $\sqrt{34+6\sqrt{41}}$	$\sqrt{37+2\sqrt{52}}$ $\sqrt{40+4\sqrt{52}}$ $\sqrt{45+6\sqrt{52}}$	$\sqrt{50+2\sqrt{65}}$ $\sqrt{53+4\sqrt{65}}$ $\sqrt{58+6\sqrt{65}}$	$\sqrt{65+2\sqrt{80}}$ $\sqrt{68+4\sqrt{80}}$ $\sqrt{73+6\sqrt{80}}$	$\sqrt{82+2\sqrt{97}}$ $\sqrt{85+4\sqrt{97}}$ $\sqrt{90+6\sqrt{97}}$			
5	$\sqrt{26+2\sqrt{50}}$ $\sqrt{29+4\sqrt{50}}$ $\sqrt{34+6\sqrt{50}}$	$\sqrt{37+2\sqrt{61}}$ $\sqrt{40+4\sqrt{61}}$ $\sqrt{45+6\sqrt{61}}$	$\sqrt{50+2\sqrt{74}}$ $\sqrt{53+4\sqrt{74}}$ $\sqrt{58+6\sqrt{74}}$	$\sqrt{65+2\sqrt{89}}$ $\sqrt{68+4\sqrt{89}}$ $\sqrt{73+6\sqrt{89}}$	$\sqrt{82+2\sqrt{106}}$ $\sqrt{85+4\sqrt{106}}$ $\sqrt{90+6\sqrt{106}}$				
6	$\sqrt{37+2\sqrt{72}}$ $\sqrt{40+4\sqrt{72}}$ $\sqrt{45+6\sqrt{72}}$	$\sqrt{50+2\sqrt{85}}$ $\sqrt{53+4\sqrt{85}}$ $\sqrt{58+6\sqrt{85}}$	$\sqrt{65+2\sqrt{100}}$ $\sqrt{68+4\sqrt{100}}$ $\sqrt{73+6\sqrt{100}}$	$\sqrt{82+2\sqrt{117}}$ $\sqrt{85+4\sqrt{117}}$ $\sqrt{90+6\sqrt{117}}$					
7	$\sqrt{50+2\sqrt{98}}$ $\sqrt{53+4\sqrt{98}}$ $\sqrt{58+6\sqrt{98}}$	$\sqrt{65+2\sqrt{113}}$ $\sqrt{68+4\sqrt{113}}$ $\sqrt{73+6\sqrt{113}}$	$\sqrt{82+2\sqrt{130}}$ $\sqrt{85+4\sqrt{130}}$ $\sqrt{90+6\sqrt{130}}$						
8	$\sqrt{65+2\sqrt{128}}$ $\sqrt{68+4\sqrt{128}}$ $\sqrt{73+6\sqrt{128}}$	$\sqrt{82+2\sqrt{145}}$ $\sqrt{85+4\sqrt{145}}$ $\sqrt{90+6\sqrt{145}}$							
9	$\sqrt{82+2\sqrt{162}}$ $\sqrt{85+4\sqrt{162}}$ $\sqrt{90+6\sqrt{162}}$								

Fractional Values of a and b.

$a=$ \ $b=$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
$\frac{1}{2}$	$\sqrt{2+\sqrt{5}}$ $\sqrt{5+2\sqrt{5}}$ $\sqrt{10+3\sqrt{5}}$	$\sqrt{5+\sqrt{17}}$ $\sqrt{8+2\sqrt{17}}$ $\sqrt{13+3\sqrt{17}}$	$\sqrt{10+\sqrt{37}}$ $\sqrt{13+2\sqrt{37}}$ $\sqrt{18+3\sqrt{37}}$	$\sqrt{17+\sqrt{65}}$ $\sqrt{20+2\sqrt{65}}$ $\sqrt{25+3\sqrt{65}}$	$\sqrt{26+\sqrt{101}}$ $\sqrt{29+2\sqrt{101}}$ $\sqrt{34+3\sqrt{101}}$
$\frac{3}{2}$	$\sqrt{5+\sqrt{25}}$ $\sqrt{8+2\sqrt{25}}$ $\sqrt{13+3\sqrt{25}}$	$\sqrt{10+\sqrt{45}}$ $\sqrt{13+2\sqrt{45}}$ $\sqrt{18+3\sqrt{45}}$	$\sqrt{17+\sqrt{73}}$ $\sqrt{20+2\sqrt{73}}$ $\sqrt{25+3\sqrt{73}}$	$\sqrt{26+\sqrt{109}}$ $\sqrt{29+2\sqrt{109}}$ $\sqrt{34+3\sqrt{109}}$	
$\frac{5}{2}$	$\sqrt{10+\sqrt{61}}$ $\sqrt{13+2\sqrt{61}}$ $\sqrt{18+3\sqrt{61}}$	$\sqrt{17+\sqrt{89}}$ $\sqrt{20+2\sqrt{89}}$ $\sqrt{25+3\sqrt{89}}$	$\sqrt{26+\sqrt{125}}$ $\sqrt{29+2\sqrt{125}}$ $\sqrt{34+3\sqrt{125}}$		
$\frac{7}{2}$	$\sqrt{17+\sqrt{113}}$ $\sqrt{20+2\sqrt{113}}$ $\sqrt{25+3\sqrt{113}}$	$\sqrt{26+\sqrt{149}}$ $\sqrt{29+2\sqrt{149}}$ $\sqrt{34+3\sqrt{149}}$			
$\frac{9}{2}$	$\sqrt{26+\sqrt{181}}$ $\sqrt{29+2\sqrt{181}}$ $\sqrt{34+3\sqrt{181}}$				

Simple Surd Values of a and b.

$a=$ \ $b=$	0	$\sqrt{5}$	$2\sqrt{5}$	$3\sqrt{5}$
$\sqrt{5}$	$\sqrt{6+2\sqrt{10}}$ $\sqrt{9+4\sqrt{10}}$ $\sqrt{14+6\sqrt{10}}$	$\sqrt{21+2\sqrt{25}}$ $\sqrt{24+4\sqrt{25}}$ $\sqrt{29+6\sqrt{25}}$	$\sqrt{46+2\sqrt{50}}$ $\sqrt{49+4\sqrt{50}}$ $\sqrt{54+6\sqrt{50}}$	$\sqrt{81+2\sqrt{85}}$ $\sqrt{84+4\sqrt{85}}$ $\sqrt{89+6\sqrt{85}}$
$2\sqrt{5}$	$\sqrt{21+2\sqrt{40}}$ $\sqrt{24+4\sqrt{40}}$ $\sqrt{29+6\sqrt{40}}$	$\sqrt{46+2\sqrt{65}}$ $\sqrt{49+4\sqrt{65}}$ $\sqrt{54+6\sqrt{65}}$	$\sqrt{81+2\sqrt{100}}$ $\sqrt{84+4\sqrt{100}}$ $\sqrt{89+6\sqrt{100}}$	
$3\sqrt{5}$	$\sqrt{46+2\sqrt{90}}$ $\sqrt{49+4\sqrt{90}}$ $\sqrt{54+6\sqrt{90}}$	$\sqrt{81+2\sqrt{125}}$ $\sqrt{84+4\sqrt{125}}$ $\sqrt{89+6\sqrt{125}}$		
$4\sqrt{5}$	$\sqrt{81+2\sqrt{160}}$ $\sqrt{84+4\sqrt{160}}$ $\sqrt{89+6\sqrt{160}}$			

Simple Surd Values of a and b.

$a=$ \ $b=$	0	$\sqrt{2}$	$2\sqrt{2}$	$3\sqrt{2}$
$\sqrt{2}$	$\sqrt{3+2\sqrt{4}}$ $\sqrt{6+4\sqrt{4}}$ $\sqrt{11+6\sqrt{4}}$	$\sqrt{9+2\sqrt{10}}$ $\sqrt{12+4\sqrt{10}}$ $\sqrt{17+6\sqrt{10}}$	$\sqrt{19+2\sqrt{20}}$ $\sqrt{22+4\sqrt{20}}$ $\sqrt{25+6\sqrt{20}}$	$\sqrt{33+2\sqrt{34}}$ $\sqrt{36+4\sqrt{34}}$ $\sqrt{41+6\sqrt{34}}$
$2\sqrt{2}$	$\sqrt{9+2\sqrt{16}}$ $\sqrt{12+4\sqrt{16}}$ $\sqrt{17+6\sqrt{16}}$	$\sqrt{19+2\sqrt{26}}$ $\sqrt{22+4\sqrt{26}}$ $\sqrt{25+6\sqrt{26}}$	$\sqrt{33+2\sqrt{40}}$ $\sqrt{36+4\sqrt{40}}$ $\sqrt{41+6\sqrt{40}}$	
$3\sqrt{2}$	$\sqrt{19+2\sqrt{36}}$ $\sqrt{22+4\sqrt{36}}$ $\sqrt{27+6\sqrt{36}}$	$\sqrt{33+2\sqrt{50}}$ $\sqrt{36+4\sqrt{50}}$ $\sqrt{41+6\sqrt{50}}$		
$4\sqrt{2}$	$\sqrt{33+2\sqrt{64}}$ $\sqrt{36+4\sqrt{64}}$ $\sqrt{41+6\sqrt{64}}$			

Simple Surd Values of a and b.

$a=$ \ $b=$	0	$\sqrt{3}$	$2\sqrt{3}$	$3\sqrt{3}$
$\sqrt{3}$	$\sqrt{4+2\sqrt{6}}$ $\sqrt{7+4\sqrt{6}}$ $\sqrt{12+6\sqrt{6}}$	$\sqrt{13+2\sqrt{15}}$ $\sqrt{16+4\sqrt{15}}$ $\sqrt{21+6\sqrt{15}}$	$\sqrt{28+2\sqrt{30}}$ $\sqrt{31+4\sqrt{30}}$ $\sqrt{36+6\sqrt{30}}$	$\sqrt{49+2\sqrt{51}}$ $\sqrt{52+4\sqrt{51}}$ $\sqrt{57+6\sqrt{51}}$
$2\sqrt{3}$	$\sqrt{13+2\sqrt{24}}$ $\sqrt{16+4\sqrt{24}}$ $\sqrt{21+6\sqrt{24}}$	$\sqrt{28+2\sqrt{39}}$ $\sqrt{31+4\sqrt{39}}$ $\sqrt{36+6\sqrt{39}}$	$\sqrt{49+2\sqrt{60}}$ $\sqrt{52+4\sqrt{60}}$ $\sqrt{57+6\sqrt{60}}$	
$3\sqrt{3}$	$\sqrt{28+2\sqrt{54}}$ $\sqrt{31+4\sqrt{54}}$ $\sqrt{36+6\sqrt{54}}$	$\sqrt{49+2\sqrt{75}}$ $\sqrt{52+4\sqrt{75}}$ $\sqrt{57+6\sqrt{75}}$		
$4\sqrt{3}$	$\sqrt{49+2\sqrt{96}}$ $\sqrt{51+4\sqrt{96}}$ $\sqrt{56+6\sqrt{96}}$			

ARITHMETIC.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

81. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

How far will a body fall in the first second on the sun, the density of the sun being 25 times that of the earth and its diameter 866400 miles?

Solution by G. B. M. ZERR, A. M., Ph. D., President of Russell College, Lebanon, Va.; and E. W. MORRELL, A. M., Professor of Mathematics, Montpelier Seminary, Montpelier, Vt.

Let G=gravity on sun, g=gravity on earth.
D=density of sun, d=density of earth.
R=radius of sun, r=radius of earth.
Then $G : g = DR : dr$. $\therefore G = gDR/dr$.
Now $g=32.2$, $D=.25d=\frac{1}{4}d$, $R=109.5r$.

$$\therefore G=\frac{32.2\times 109.5}{4}=881.475.$$

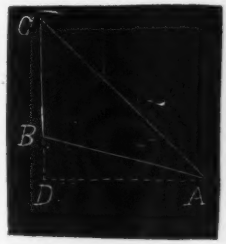
$\frac{1}{2}G=440.7375$ feet, the distance a body will fall the first second.

82. Proposed by CHAS. C. CROSS, Laytonsville, Md.

Two men, A and B, started from the same point at the same time; A traveled southeast for 10 hours and at the rate of 10 miles per hour, and B due south for the same time, going 6 miles per hour; they then turned and traveled directly towards each other at the same rates respectively, till they met. How far did each man travel?

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; G. B. M. ZERR, A. M., Ph. D., President of Russell College, Lebanon, Va.; P. S. BERG, Larimore, N. D.; and H. C. WILKES, Skull Run, Va.

Let C be the starting point; CA=the distance A traveled southeast, and CB=the distance B traveled south. Then $CA=100$ miles, and $CB=60$ miles. Now draw AD perpendicular to CB produced to D. As $\angle D$=a right angle, and $\angle C=45^\circ$, then $CD=AD$.



Then $2\overline{AD}^2=\overline{AC}^2=100^2$; whence $AD=50\sqrt{2}$, and $BD=50\sqrt{2}-60$. $\therefore$ From the right triangle ADB,

$$AB=\sqrt{(50\sqrt{2})^2+(50\sqrt{2}-60)^2}=\sqrt{13600-6000\sqrt{2}}$$

$$=71.517261+ \text{ miles.}$$

A and B together travel 16 miles per hour, and the time required, until they meet in traveling AB, is $\frac{1}{16}$ of AB=4.469828+ hours. Therefore, A traveled 44.69828+ miles and B, 26.81897+ miles of the distance AB.

$\therefore$ The total distance traveled by A is 144.69828+ miles, and by B, 86.81897+ miles.

This problem was also solved by *F. R. HONEY, C. A. JONES*, and *E. W. MORRELL*.

Solutions of problem 80 were received from G. B. M. Zerr, P. S. Berg, E. W. Morrell, F. R. Honey, and H. C. Wilkes.

NOTE. Hon. Josiah H. Drummond says, in reference to problem 78: "How can you make $986\times3+569=3905$? The question is erroneously enunciated or erroneously solved, or both."

If we assume that the problem is correctly stated, then certainly the published solution is not the solution of the problem. The following is an algebraic statement of the problem as proposed:

Let x=number of cows. Then $3x+569$=number of horses. Let y=number of sheep. Then $4y-126$=number of cows. Hence, $x=4y-126$, $3x=12y-378$, and $3x+569=12y-378+569=12y+191$, the number of horses expressed in terms of the number of sheep. Hence, y, the number of sheep, $+4y-126$, the number cows, $+12y+191$, the number of horses, or $17y+65$=total number=5169. Solving this equation, we do not obtain integral results. If 9 were changed to 5, then y would be integral, and the problem possible. We failed to find this problem in Brooks' Higher Arithmetic. EDITOR.

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

72. Proposed by CHAS. C. CROSS, Laytonsville, Md.

Prove that $\dfrac{2\sqrt{2+\sqrt{3}}}{4+\sqrt{6-\sqrt{2}}}=\sqrt{6}-\sqrt{2}+\sqrt{3}-2$, when reduced to its lowest terms.

I. Solution by JOSIAH H. DRUMMOND, Portland, Maine.

$$\frac{2\sqrt{2+\sqrt{3}}}{4+\sqrt{6-\sqrt{2}}}=\frac{\sqrt{2}\sqrt{4+2\sqrt{3}}}{4+\sqrt{2}(\sqrt{3}-1)},\quad =\frac{1+\sqrt{3}}{2\sqrt{2}+\sqrt{3}-1},$$

$$=\frac{(1+\sqrt{3})(2\sqrt{2}-\sqrt{3}+1)}{2(2+\sqrt{3})},\quad =\frac{(2-\sqrt{3})(1+\sqrt{3})(2\sqrt{2}-\sqrt{3}+1)}{2},$$

$$=\frac{(\sqrt{3}-1)(2\sqrt{2}-\sqrt{3}+1)}{2},\quad =\sqrt{6}-\sqrt{2}+\sqrt{3}-2.$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Russell College, Lebanon, Va.; and P. S. BERG, Principal of Schools, Larimore, N. D.

$$\frac{2\sqrt{2+\sqrt{3}}}{4+\sqrt{6-\sqrt{2}}}=\frac{\sqrt{6}+\sqrt{2}}{4+\sqrt{6}-\sqrt{2}},\quad =\frac{(\sqrt{6}+\sqrt{2})(4+\sqrt{6}+\sqrt{2})}{(4+\sqrt{6})^2-2},$$

$$=\frac{\sqrt{6}+\sqrt{3}+\sqrt{2}+2}{5+2\sqrt{6}}, \quad =\frac{(\sqrt{6}+\sqrt{3}+\sqrt{2}+2)(5-2\sqrt{6})}{25-24},$$

$$=\sqrt{6}-\sqrt{2}+\sqrt{3}-2.$$

Also solved by *COOPER D. SCHMITT* and the *PROPOSER*.

73. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, Russell College, Lebanon, Va.

Find the worth of each of five persons, A, B, C, D, and E, knowing, 1st, that when A's worth is added to a times what B, C, D, and E are worth, it is equal to m; 2nd, when B's worth is added to b times what A, C, D, and E are worth, it is equal to n; 3rd, when C's worth is added to c times what A, B, D, and E are worth, it is equal to p; 4th, when D's worth is added to d times what A, B, C, and E are worth, it is equal to q; 5th, when E's worth is added to e times what A, B, C, and D are worth, it is equal to r.

I. Solution by the PROPOSER.

Let x, y, z, u, v be the worth of A, B, C, D, and E, respectively. Then

$$\begin{aligned} x+a(y+z+u+v)&=m,\\ y+b(x+z+u+v)&=n,\\ z+c(x+y+u+v)&=p,\\ u+d(x+y+z+v)&=q,\\ v+e(x+y+z+u)&=r. \end{aligned}$$

Let $x+y+z+u+v=s$; then $x+a(s-x)=m$.

$\therefore\ x=(m-as)/(1-a)$.....(1). Similarly, $y=(n-bs)/(1-b)$.....(2),

$z=(p-cs)/(1-c)$.....(3), $u=(q-ds)/(1-d)$.....(4),

$v=(r-es)/(1-e)$.....(5).

Adding (1), (2), (3), (4), and (5), we get

$$s=\frac{m-as}{1-a}+\frac{n-bs}{1-b}+\frac{p-cs}{1-c}+\frac{q-ds}{1-d}+\frac{r-es}{1-e}.$$

$$\therefore\ s=\frac{\left\{\frac{m}{1-a}+\frac{n}{1-b}+\frac{p}{1-c}+\frac{q}{1-d}+\frac{r}{1-e}\right\}}{\left\{1+\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}+\frac{d}{1-d}+\frac{e}{1-e}\right\}}.$$

This value of s in (1), (2), (3), (4), (5) gives x, y, z, u, v.

II. Solution by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; and Professor CHAS. C. CROSS, Laytonsville, Md.

By the conditions we have at once the five equations ; letting x, y, z, u, t, $=A$, B, C, D, E, and F's shares, respectively :

$$\begin{aligned} x+ay+az+au+at&=m,\\ bx+y+by+bu+bt&=n,\\ cx+cy+z+cu+ct&=p,\\ dx+dy+dz+u+dt&=q,\\ ex+ev+ez+eu+t&=r. \end{aligned}$$

Hence by Determinants,

$$x=\begin{vmatrix} m & a & a & a & a \\ n & 1 & b & b & b \\ p & c & 1 & c & c \\ q & d & d & 1 & d \\ r & e & e & e & 1 \end{vmatrix} \div \begin{vmatrix} 1 & a & a & a & a \\ b & 1 & b & b & b \\ c & c & 1 & c & c \\ d & d & d & 1 & d \\ e & e & e & e & 1 \end{vmatrix},$$

$$y=\begin{vmatrix} 1 & m & a & a & a \\ b & n & b & b & b \\ c & p & 1 & c & c \\ d & q & d & 1 & d \\ e & r & e & e & 1 \end{vmatrix} \div \begin{vmatrix} 1 & a & a & a & a \\ b & 1 & b & b & b \\ c & c & 1 & c & c \\ d & d & d & 1 & d \\ e & e & e & e & 1 \end{vmatrix},$$

and so with z, u, and t, each determinant possessing 120 terms, when expanded.

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

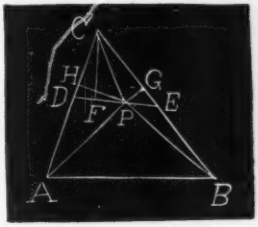
SOLUTIONS OF PROBLEMS.

76. Proposed by L. B. FRAKER, Bowling Green, Ohio.

Lines run from a point, P, within a triangular piece of land to the angles A, B, and C are 91, 102, and 80 rods, respectively; and a line 78 rods in length passing through the point, P, and terminating in the sides AC and BC cuts off 3024 square rods adjacent to angle C. Required the dimensions of the land.

Solution by CHAS. C. CROSS, Laytonsville, Md.

Let ABC be the required triangle, $AP=a=91$ rods, $BP=b=102$ rods, $CP=c=80$ rods, $DE=d=78$ rods, the line drawn through P and cutting off 3024 square rods adjacent to the angle C, $CD=x$, $CE=y$, $PE=z$, and area of triangle $DEC=k$.



Draw the perpendiculars PG and PH, from the point P to the sides BC and AC respectively, and draw CF perpendicular to DE. Then

$$x^2=y^2+d^2\pm 2d\times FE, \text{ whence}$$

$$FE=\frac{y^2+d^2-x^2}{2d}. \quad \text{Also } FE=\pm\sqrt{y^2-\frac{4k^2}{d^2}}.$$

$$\text{Hence, } \mp\frac{y^2+d^2-x^2}{2d}=\pm\sqrt{y^2-\frac{4k^2}{d^2}},$$

whence $-(x^2-y^2)^2+2d^2(x^2+y^2)=16k^2+d^4$, an equation containing two unknown quantities. Hence since no other conditions are given by which x or y can be found, it follows that the problem is indeterminate.

By trial, we find that $x=90$ and $y=84$ satisfies the above equation.

Hence, these values furnish a solution, in positive integers, of the problem. Then

$$PF=\sqrt{c^2-\frac{4k^2}{d^2}}=19\tfrac{9}{13} \text{ rods, and } FE=\sqrt{y^2-\frac{4k^2}{d^2}}=32\tfrac{4}{13} \text{ rods.}$$

Hence, $z=PF+FE=52$ rods.

$$BG=\frac{b^2+BC^2-c^2}{2BC} \text{ and } CG=\frac{c^2+y^2-z^2}{2y}. \quad \text{But } BG+CG=BC. \quad \text{Hence,}$$

$$BC=\frac{c^2+y^2-z^2\pm\sqrt{4(b^2-c^2)y^2+(c^2+y^2-z^2)^2}}{2y}=154 \text{ rods.}$$

By similar reasoning with the triangles APC and DPC, we find that $AC=165$ rods.

$$\cos ACB=(90^2+84^2-18^2)\div 2.90.84=\tfrac{3}{5}.$$

$$AB=\sqrt{AC^2+BC^2-2AC.BC\times\cos ACB}=143 \text{ rods.}$$

Hence, the dimensions of the field are $AB=143$ rods, $BC=154$ rods, and $AC=165$ rods.

This problem was proved indeterminate by A. H. Bell.

77. Proposed by CHAS. C. CROSS, Laytonsville, Md.

A line is drawn perpendicular to BC, of the triangle ABC, whose sides are $BC=a$, $CA=b$, and $AB=c$, through A to D, a distance d, (d being equal to or greater than $a+b$); from D a line is drawn to E, a distance e, (e being equal to or greater than $a+b+c$) on BC extended. Required the area of the ellipse which is isogonal conjugate to the straight line DE with respect to the triangle ABC.

I. Solution by G. B. M. ZERR, A. M., Ph. D., President of Russell College, Lebanon, Va.

Using trilinear coördinates and letting F be the point where $AD=d$ cuts BC, we get $DF=(d-b\sin C)$, $EF=\sqrt{e^2-(d-b\sin C)^2}=f$.

$\therefore$ The coördinates of D, E are respectively,

$$\{-(d-b\sin C),\ e\cos C,\ e\cos B\} \text{ and}$$
$$\{0,\ -(f-b\cos C)\sin C,\ -(f+c\cos B)\sin B\}.$$

Let $l=e\{(f-b\cos C)\sin C\cos C-(f+c\cos B)\sin B\cos B\}$,

$m=-(d-b\sin C)(f+c\cos B)\sin B$, and

$n=(d-b\sin C)(f-b\cos C)\sin C$.

Then $l\alpha+m\beta+n\gamma=0$, is the equation to DE, and $l\beta\gamma+m\gamma\alpha+n\alpha\beta=0$, is the equation to the ellipse isogonal conjugate to DE.

Let B be the origin, BC, BA the axes of (x, y).

Then $\alpha=y\sin B$, $\gamma=x\sin B$, $\beta=(ac\sin B-a\alpha-c\gamma)/b$.

$\therefore$ $\beta=\sin B(ac-ay-cx)/b$.

Substituting these values of α, β, γ the equation to the ellipse becomes,

$$clx^2+any^2+(al+cn-bm)xy-aclx-acny=0.$$

Let $\Delta=\dfrac{a^2c^2ln(2-cn-ae)}{4acln-(al+cn-bm)^2}$ be the discriminant of the ellipse.

The two values of z in the equation,

$$z^2+\frac{16(cl+an)\Delta}{\{4acln-(al+cn-bm)^2\}^2}\ z-\frac{64\Delta^2}{\{4acln-(al+cn-bm)^2\}^3}=0,$$

give the values of the squares of the semi-axes.

$\therefore$ Area of ellipse

$$=\frac{8\pi\Delta}{\{4acln-(al+cn-bm)^2\}^{\frac{1}{2}}}=\frac{8\pi a^2c^2ln(2-cn-al)}{\{4acln-(al+cn-bm)^2\}^{\frac{5}{2}}}.$$

II. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio University, Athens, Ohio.

The coördinates of D are $(d, -\beta_1, -\gamma_1)$, and of E, $(0, \beta_2, -\gamma_2)$, and we then have,

$$e^2=(abc)/(4\Delta^2)\{a(\beta_1+\beta_2)(\gamma_2-\gamma_1)+bd(\gamma_1-\gamma_2)+cd(\beta_1+\beta_2)\} \quad \ldots\ldots\ldots\ldots (1).$$

The equation to the perpendicular to BC through A is

$$\gamma\cos C-a\cos A=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2),$$

and this, passing through D, gives

$$\gamma_1\cos C+d\cos A=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3).$$

We have the constant relation

$$a\alpha+b\beta+c\gamma=2\Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (4),$$

and this being satisfied by the coördinates of D and E,

$$ad-b\beta_1-c\gamma_1=2\Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (5).$$

$$b\beta_2+c\gamma_2=2\Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (6).$$

The equation to DE is

$$a(\beta_1\gamma_2+\beta_2\gamma_1)+\beta d\gamma_2+\gamma d\beta_2=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (7),$$

the isogonal conjugate of which is

$$\beta\gamma(\beta_1\gamma_2+\beta_2\gamma_1)+\alpha\gamma d\gamma_2+\alpha\beta d\beta_2=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (8),$$

which by the problem is an ellipse.

The area of (8) is expressed by

$$K=2\pi\Delta abc\left\{\begin{vmatrix} 0, & \frac{1}{2}d\beta_2, & \frac{1}{2}d\gamma_2 \\ \frac{1}{2}d\beta_2, & 0, & \frac{1}{2}(\beta_1\gamma_2+\beta_2\gamma_1) \\ \frac{1}{2}d\gamma_2, & \frac{1}{2}(\beta_1\gamma_2+\beta_2\gamma_1), & 0 \end{vmatrix} \div \right.$$

$$\left.\begin{vmatrix} 0, & \frac{1}{2}d\beta_2, & \frac{1}{2}d\gamma_2, & -a \\ \frac{1}{2}d\beta_2, & 0, & \frac{1}{2}(\beta_1\gamma_2+\beta_2\gamma_1), & -b \\ \frac{1}{2}d\gamma_2, & \frac{1}{2}(\beta_1\gamma_2+\beta_2\gamma_1), & 0, & -c \\ a, & b, & c, & 0 \end{vmatrix}^{\frac{3}{2}}\right\} \quad \ldots\ldots (9).$$

β_1, γ_1 are determined by (3) and (5), and then β_2 and γ_2 from (1) and (6), giving K in terms of d and elements of the triangle of reference.

It is not obvious how much of a reduction (9) admits, and I have not attempted any.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

59. Proposed by MOSES COBB STEVENS, M. A., Department of Mathematics, Purdue University, Lafayette, Ind.

Solve $n\frac{d^2y}{dx^2}(x^2+y^2)^{\frac{1}{2}}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$.

[From *Forsyth's Differential Equations.*]

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

Let $y/x=z$, $d^2y/dx^2=y/x$; then the given equation becomes

$$nv\sqrt{1+z^2}=\{1+p^2\}^{\frac{3}{2}},$$

$$\text{or } v=\frac{\{1+p^2\}^{\frac{3}{2}}}{n\sqrt{1+z^2}},\ =(p-z)\frac{dp}{dz}\ldots\ldots\ldots\ldots\ldots\ldots(A).$$

Assuming $p=(z-t)/(1+zt)$, (A) reduces to

$$\frac{t(dt/dz)}{[1+t^2][t+(1/n)\sqrt{1+t^2}]}-\frac{1}{1+z^2}=0\ldots\ldots\ldots\ldots\ldots(B),$$

in which the variables are separated.

II. Solution by G. B. M. ZERR, M. A., Ph D., President and Professor of Mathematics, Russell College, Lebanon, Va.

Let $x=r\cos\theta$, $y=r\sin\theta$, then the equation becomes,

$$nr^3+2nr\left(\frac{dr}{d\theta}\right)^2-nr^2\frac{d^2r}{d\theta^2}=\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{\frac{3}{2}}\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Let $\frac{dr}{d\theta}=p$, so that $\frac{d^2r}{d\theta^2}=p\frac{dp}{dr}$;

$\therefore$ (1) becomes, $nr^3+2nrp^2-nr^2p\frac{dp}{dr}=(r^2+p^2)^{\frac{3}{2}}$.

Let $p=r/v$, so that $\frac{dp}{dr}=\frac{1}{v}-\frac{r}{v^2}\frac{dv}{dr}$;

$\therefore$ (2) becomes, $nv(1+v^2)dr+nrdv=(1+v^2)^{\frac{3}{2}}dr$.

$$\therefore\ dr/r=-\frac{n\,dv}{(1+v^2)\{(1+v^2)^{\frac{1}{2}}-nv\}}.$$

$$\therefore\ \log r=\log\left\{\frac{(1+v^2)^{\frac{1}{2}}}{(1+v^2)^{\frac{1}{2}}-nv}\right\}+\log A.$$

$$\therefore\ r=\frac{A(1+v^2)^{\frac{1}{2}}}{(1+v^2)^{\frac{1}{2}}-nv},\quad =\frac{A(r^2+p^2)^{\frac{1}{2}}}{(r^2+p^2)^{\frac{1}{2}}-nr}.$$

$$\therefore\ r\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{\frac{1}{2}}-nr^2=A\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{\frac{1}{2}}.$$

$$\therefore\ d\theta=\pm\frac{(r-A)dr}{r\sqrt{n^2r^2-(r-A)^2}},\quad =\pm\frac{(r-A)dr}{nr^2\sqrt{1-\{(r-A)/(nr)\}^2}}.$$

$$\therefore\ \theta=\pm\frac{1}{\sqrt{n^2-1}}\log[(n^2-1)r+A+\sqrt{(n^2-1)\{r^2n^2-(r-A)^2\}}\,]$$
$$\mp\sin^{-1}\left(\frac{r-A}{nr}\right)+B.$$

$$\therefore\ \tan^{-1}(y/x)\pm\sin^{-1}\left(\frac{\sqrt{x^2+y^2}-A}{n\sqrt{x^2+y^2}}\right)-B,=\pm\frac{1}{\sqrt{n^2-1}}\log[(n^2-1)\sqrt{x^2+y^2}$$
$$+A+\sqrt{(n^2-1)\{n^2(x^2+y^2)-(\sqrt{x^2+y^2}-A)^2\}}\,],$$

where A and B are constants of integration.

60. Proposed by SETH PRATT, C. E., Assyria, Mich.

To remove $(1/a)$th of the volume of a sphere of a given radius by a conical hole, whose axis is the axis of the sphere, and whose vertex is at the surface of the sphere. Required the height of the cone and the diameter of its base.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let O be the center of the sphere, ABC the cone, $AO=r$, $DE=x$, $AD=y$; then the volume of spherical segment

$$ABE=\frac{\pi x^2}{3}(3r-x),$$

and that of the cone

$$ABC=\frac{\pi y^2(2r-x)}{3};$$

by condition, therefore,

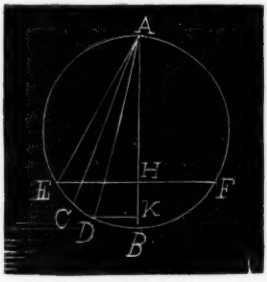
$\frac{\pi x^2}{3}(3r-x)+\frac{\pi y^2}{3}(2r-x)=(1/a).\frac{4}{3}\pi r^3$. Substituting $y^2=2rx-x^2$, we obtain the final equation, $x^2-4rx=-(4r^2/a)$, whence $x=2r\{1-[1-(1/a)]^{\frac{1}{2}}\}$.

$\therefore$ Height of cone $2r-x=2r[1-(1/a)]^{\frac{1}{2}}$, and $y=2r\{[1-(1/a)]^{\frac{1}{2}}-[1-(1/a)]\}^{\frac{1}{2}}$.

II. Solution by C. W. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Mass.

Figure shows section of sphere through axis, with $AEBF$ as section of hole.

Let $h=AH$=height of cone ; $b=FH$=radius of base ; $dA=ACD$=elements of area AEB ; $x(=DK)$ is perpendicular to AB; $\theta=\angle BAD$.



Then $dA=\frac{1}{2}\overline{AD}^2 d\theta=2r^2\cos^2\theta d\theta$.

Center of gravity of ADC is at distance $2x/3$ from AB. The element of volume found by revolving ADC about AB, or $dV=2\pi\times(2x/3)\times 2r^2\cos^2\theta d\theta$.

But $x=AD\sin b=2r\cos\theta\sin\theta$.

$\therefore\ dV=(16\pi/3)r^3\cos^3\theta\sin\theta d\theta$;

$$\cos\angle EAH=\frac{AH}{AE}=\frac{h}{\sqrt{2rh}}=\sqrt{\frac{h}{2r}}.$$

$$\therefore \text{ Volume } AEBF=\frac{16\pi r^3}{3}\int_0^{\cos^{-1}\sqrt{(h/2r)}}\cos^3\theta\sin\theta d\theta,=(\pi r/3)(4r^2-h^2),$$

which equals $(1/a)$th of volume of sphere, or $4\pi r^3/3a$.

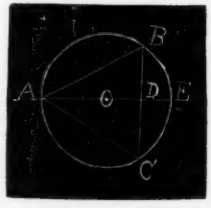
$\therefore\ h=2r\sqrt{[1-(1/a)]}$.

$$\text{Diameter}=2b=2\sqrt{h(2r-h)}=4r\sqrt{1-(1/a)[1-\sqrt{1-(1/a)}]}.$$

Volume $AEBF$ can be as easily found by geometry without the use of calculus.

III. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, Russell College, Lebanon, Va.

Let $AO=r$, $DO=y$, $DB=x$. Volume of cone $ABC=\frac{1}{3}\pi(r+y)x^2$; volume of segment $BDCE=\frac{1}{3}\pi(r-y)^2(2r+y)$.



$\therefore\ \frac{1}{3}\pi(r+y)x^2+\frac{1}{3}\pi(r-y)^2(2r+y)=(4\pi r^3/3a)$, but $x^2=r^2-y^2$.

$\therefore\ (r+y)(r^2-y^2)+(r-y)^2(2r+y)=(4r^3/a)$.

$\therefore\ y^2+2ry=(3ar^2-4r^2)/a$.

$$\therefore\ y=\pm 2r\sqrt{\frac{a-1}{a}}-r.$$

The plus sign alone is admissible. $\therefore y=2r\sqrt{\frac{a-1}{a}}-r.$

$\therefore$ Altitude $=r+y$, $=2r\sqrt{\frac{a-1}{a}}$; $2x$=diameter of base$=2\sqrt{r^2-y^2}$.

$$2x=2r\sqrt{\sqrt{\frac{a-1}{a}}-\frac{a-1}{a}},\ =2r\sqrt[4]{\frac{a-1}{a}}\sqrt{\frac{\sqrt{a-1}}{a}}.$$

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

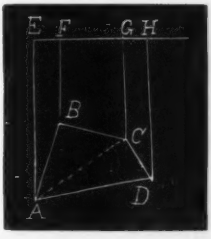
SOLUTIONS OF PROBLEMS.

50. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A plane quadrilateral $ABCD$ in the vertical wall of a cistern, filled with water, has its four vertices A, B, C, D at the distances 10 feet, 4 feet, 5 feet, and 7 feet respectively, from the surface of the water. The projections of AB, BC, CD upon the surface are respectively 2 feet, 3 feet, and 1 foot. Find the pressure of the water upon the quadrilateral, and the position of the center of mean pressure.

Solution by J. C. NAGLE, A. M., M. C. E., Professor of Civil Engineering in the State Agricultural and Mechanical College, College Station, Texas, and the PROPOSER.

In the figure, $AE=10$, $BF=4$, $CG=5$, $DH=7$, $EF=2$, $FG=3$, $GH=1$. The coördinates of the vertices A, B, C, and D with respect to EH and EA as axes are respectively, (10, 0), (2, 4), (5, 5), and (6, 7).



Hence, the area of $ABCD$

$$=\tfrac{1}{2}\left[\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}+\begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix}+\begin{vmatrix} x_3 & x_4 \\ y_3 & y_4 \end{vmatrix}\right]=17\tfrac{1}{2};$$

area of triangle $ABC=10$; and area of $ACD=7\frac{1}{2}$.

Distance of center of gravity of $\triangle ABC$ from $EH=\frac{1}{3}(10+4+5)=6\frac{1}{3}$, and distance of center of gravity of $\triangle ACD$ from $EH=\frac{1}{3}(10+4+7)=7\frac{1}{3}$. Denoting the distance of center of gravity of $ABCD$ by z, we have $17\frac{1}{2}z=10\times 6\frac{1}{3}+7\frac{1}{2}\times 7\frac{1}{3}$. Hence, $z=6\frac{1}{2}\frac{6}{1}$. Hence, pressure of water upon $ABCD=17\frac{1}{2}\times 6\frac{1}{2}\frac{6}{1}w=3\frac{5}{2}\frac{5}{0}w$, w denoting the weight of a cubic foot of water.

For $u=62\frac{1}{2}$ pounds, we find the pressure to be $110934\frac{3}{4}$ pounds.

Let AD represent the surface of the water, $ABCD$ a rectangle, BCE a right triangle, $AE=a$, $CD=b$, $AD=c$. Then, omitting w, the moment of the

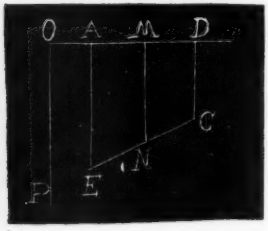
pressure upon $ABCD$ with respect to $AD = c\int_0^b x^2 dx = \frac{1}{3}b^3c$, and the moment of the pressure upon $BCE = \frac{c}{a-b}\int_0^{a-b}(a-b-x)(x+b)^2 dx = 1\frac{1}{12}c(a-b)(a^2+2ab+3b^2)$.

Adding, we find the moment of the pressure upon the trapezoid $AECD$ with respect to $AD = 1\frac{1}{12}c(a+b)(a^2+b^2)$.

For $ABFE$, $a=10$, $b=4$, $c=2$; $\therefore$ Moment $= 270\frac{2}{3}$.
For $FBCG$, $a=4$, $b=5$, $c=3$; $\therefore$ Moment $= 92\frac{1}{4}$.
For $GCDH$, $a=5$, $b=7$, $c=1$; $\therefore$ Moment $= 74$.
For $EADH$, $a=10$, $b=7$, $c=6$; $\therefore$ Moment $= 1266\frac{1}{2}$.

Adding the moments of the first three and then subtracting the sum from the moment of the fourth, we get the moment of $ABCD = 829\frac{7}{12}$. Therefore, distance of the center of pressure of $ABCD$ from $EH = 829\frac{7}{12} \div \frac{3}{5}\frac{5}{5} = 7\frac{3}{28}\frac{3}{4}$.

Let $AECD$ represent a trapezoid with right angles at A and D, AD the surface of the water, OP a perpendicular, and MN a perpendicular to OD ; $AE=a$, $CD=b$, $AD=c$, $OA=h$, $MN=y$, $AM=x$.



$\therefore$ Moment of pressure upon $AECD$ with respect to OP

$$= \tfrac{1}{2}\int_o^c y^2(x+h)dx, \text{ where } y = \frac{c}{a-b}(a-b-x).$$

Substituting, we get for the moment of $AECD$ with respect to AD the expression $2\frac{1}{24}c[c(a^2+2ab+3b^2)+4h(a^2+ab+b^2)]$.

For $ABFE$, $a=10$, $b=4$, $c=2$, $h=0$; $\therefore$ Moment $= 38$.
For $BCGF$, $a=4$, $b=5$, $c=3$, $h=2$; $\therefore$ Moment $= 110\frac{1}{8}$.
For $CDHG$, $a=5$, $b=7$, $c=1$, $h=5$; $\therefore$ Moment $= 100\frac{1}{12}$.
For $ADHE$, $a=10$, $b=7$, $c=6$, $h=0$; $\therefore$ Moment $= 580\frac{1}{2}$.

Subtracting the sum of the first three from the last, we find for the moment of $ABCD$ with respect to AE, $330\frac{1}{2}\frac{1}{4}$.

$\therefore$ Distance of the center of pressure from $AE = 330\frac{1}{2}\frac{1}{4} \div \frac{3}{5}\frac{5}{5} = 2\frac{3}{5}\frac{2}{8}\frac{5}{4}\frac{1}{0}$.

And thus the position of the center of pressure is fully determined.

51. Proposed by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

"Swift of foot was Hiawatha.
He could shoot an arrow from him
And run forward with such fleetness
That the arrow fell behind him!
Strong of arm was Hiawatha;
He could shoot ten arrows upward
Shoot them with such strength and swiftness
That the tenth had left the bowstring
Ere the first to earth had fallen." *Longfellow.*

Assuming Hiawatha to have been able to shoot an arrow every second and to have aimed when not shooting vertically so that the arrow might have the longest range; what was Hiawatha's time in a hundred yards?

I. Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University, Miss.

An arrow rises $4\frac{1}{2}$ seconds when shot vertically, and therefore, the initial velocity which Hiawatha is able to impart to an arrow is $\frac{9}{2}g$ feet per second.

The angle of elevation for the longest range is 45°, and therefore, the horizontal component of the velocity of the arrow is $\frac{1}{4}(9\sqrt{2})g$. This being Hiawatha's speed, his time for 100 yards is a very little less than 3 seconds.

In the above it has been assumed that Hiawatha ran the whole distance at a uniform rate. The range is much more than a hundred yards.

II. Solution by S. ELMER SLOCUM, Union College, Schenectady, N. Y.; J. P. BURDETT, Class '97, Dickinson College, Carlisle, Penn.; and E. W. MORRELL, A. M., Professor of Mathematics, Montpelier Seminary, Montpelier, Vt.

Let t=time of flight when the arrows are shot vertically upward, and u be the initial velocity. Then $t=2u/g$, and $u=\frac{1}{2}gt=144$ feet per second.

The range of a projectile is $u^2\sin 2\theta/g$, and since the greatest value of $\sin 2\theta$ is 1, the maximum range is u^2/g.

$\therefore$ Range$=u^2/g=648$ feet. Time of flight for projectile is $2u\sin\theta/g$ $=6.363$ seconds.

$\therefore$ Velocity$=648\div 6.363=101.8+$ feet per second.

Time for 100 yards$=(300\div 101.8+)=2.94$ seconds.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

52. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

A straight line of length a is divided into three parts by two points taken at random; find the chance that no part is greater than b. [From *Hall and Knight's Higher Algebra.*]

I. Solution by HENRY HEATON, M. Sc., Atlantic, Iowa.

There are two cases. I, when $b>\frac{1}{3}a$ and $<\frac{1}{2}a$, and II, when $b>\frac{1}{2}a$ and $<a$.

Case I. Let AB represent the line a. Let P be the position of the first point, and let $AP=x$. Lay off PC and BD each$=b$. Then the favorable positions for the second point lie between C and D. $DC=x+2b-a$. The limits of x are $a-2b$ and b.

Hence the required chance is $P_1=\frac{1}{a^2}\int_{a-2b}^{b}(x+2b-a)dx=\frac{(3b-a)^2}{2a^2}$.

Case II. In this case the limits of x are 0 and $a-b$.

Hence, $P_2 = \int_0^{a-b} (x+2b-a)dx = \dfrac{(3b-a)(a-b)}{2a^2}$.

Corollary. When $b=\frac{1}{2}a$, $P_1=P_2=\frac{1}{8}$.

II. Solution by J. O. **MAHONEY**, B. E., M. S., Graduate Fellow and Assistant in Mathematics, Vanderbilt University, Nashville, Tenn.

Let AB be the straight line of length a, and let the random points P, Q be at distances x, y from A, so that $AP=x$, $AQ=y$, and $PQ=a-x-y$. In favorable cases we must have $x<b$, $y<b$, and $a-x-y<b$; and in possible cases $x+y<a$.

$A \qquad P \qquad Q \qquad B$

Construct the right-angled triangle ABC where $AB=AC=a$. With A as origin and AC and AB as axes construct the lines MN, LH, and RS, whose equations are $y=b$, $x=b$, and $x+y=a-b$, respectively. (1) When $b>\frac{1}{2}a$, the favorable cases will be restricted to the area $MNHLSR$ in Fig. 1, and the required chance is $1-3[(a-b)/a]^2$. (2) When $b<\frac{1}{2}a$, the favorable cases will be restricted to the area 123 in Fig. 2. This is a right-angled isosceles triagle a side of which is $AM-SL=b-(a-2b)=3b-a$. Therefore, the required chance is $[(3b-a)/a]^2$.

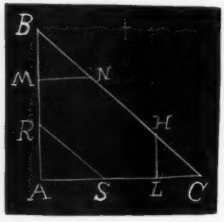


Fig. 1.

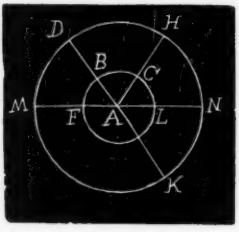
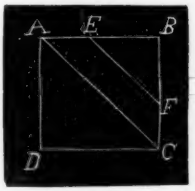


Fig. 2.

III. Solution by G. B. M. **ZERR**, M. A., Ph D., President and Professor of Mathematics, Russell College, Lebanon, Va.

Let $ABCD$ be a square side a, and take $AE=CF=b$. The coördinates of a point taken at random in $ABCD$ are the distances of two such points from one end of the line.

Without restriction the point might fall anywhere upon ABC, but the condition confines it to the triangle EBF.



$$\therefore\ p = \frac{EBF}{ABC} = \frac{(a-b)^2}{a^2} = \left(\frac{a-b}{a}\right)^2.$$

$$\text{Otherwise } p = \frac{\int_b^a \int_0^{y-b} dydx}{\int_0^a \int_0^y dydx} = \left(\frac{a-b}{a}\right)^2.$$

Professors Scheffer and Zerr should have received credit in the last number of the MONTHLY for solving problem 50, and Professor Henry Heaton should have received credit for solving problem 51.

No solution of problem 53 has yet been received.

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

45. Proposed by EDWARD R. ROBBINS, Master in Mathematics and Physics, Lawrenceville School, Lawrenceville, New Jersey.

Required several numbers each of which, when divided by 10 leaves a remainder 9; by 9 leaves 8; by 8 leaves 7; by 7 leaves 6; and so on. Also find the least such number which, when divided by 28 leaves 27; by 27 leaves 26; by 26 leaves 25; by 25 leaves 24, *et cetera ad unum.*

I. Solution by M. W. HASKELL, A. M., Ph. D., Department of Mathematics, University of California, Berkley, Cal.; NELSON L. RORAY, Professor of Mathematics in South Jersey Institute, Bridgeton, N. J.; A. H. BELL, Hillsboro, Ill., and H. C. WILKES, Skull Run, W. Va.

The problem can be also stated as follows: Required several numbers each of which, divided by 10, 9, 8, and so on, leaves a remainder (-1).

If then L be the least common multiple of 10, 9, 8 and so on, all numbers of the form $kL-1$, where k is any integer, will have the required character.

Now the least common multiple of 10, 9, 8, ...2, 1 is 2520. The required numbers are then $(2520\,k-1)$ *e. g.*, 2519, 5039, 7559, 10079, etc.

The second problem is solved in exactly the same way. The least common multiple of 28, 27, 26, ...2, 1 is 80313433200. So the required number is one less, or 80313433199.

II. Solution by the PROPOSER.

One less than the product of any number of factors will be divisible by any of the factors, or products of any or all of them, with a remainder one less than the divisor. Because $ab^2c^2d^3-1$ divided by acd gives b^2cd^2-1 for quotient and $acd-1$ for remainder. Thus, the different factors occurring in the natural numbers 1, 2, 3, etc., to 10, are (1.2.2.2.3.3.5.7), one less than the product of which is 2519, which leaves remainders less by unity than the divisors when divided by numbers 1, 2, 3, ...10. All multiples (diminished by one) of the continued product of these factors will satisfy the same demands of the problem, to-wit: 7559, 10079, 12599, etc., etc., *ad libitum.*

The factors occuring in numbers 1.....28 are (1.2.2.2.2.3.3.3.5 5.7.11. 13.17.19.23) and one less than their continued product gives 80313433199, the number required.

NOTE. Of course the same numbers will accomodate 5 and 6; 9 and 10; 11 and 12; 13, 14, and 15; 17 and 18; 19, 20, 21, and 22; 23 and 24; 25 and 26; 27 and 28; and so on.

III. Solution by JOSIAH H. DRUMMOND, Portland, Maine.

I. $10a+9$ answers the first condition; multiply this by 9 and add 8, and

we have $90a+89$; proceeding in the same manner we finally have $3,628,800a+3,628,799$, in which a may be zero or any number.

II. Or, in the process as above, we may leave out factors of numbers already used and we reach the result $2520a+2519$, in which a may be zero or any number ; if a=zero, we have 2519, the smallest number that will answer the conditions of the first question.

III. It is manifest that if we take 1 from a number divisible by all the given divisors, the remainder when divided by those divisors will always leave a remainder one less than the divisor. Hence the least common multiple of the given divisors, less 1, is the number required. Hence, omitting the common factors in the second part of the question, we have $28.27.26.25.23.22.19.17....1 = 80,313,433,199$, the number required.

IV. Solution by W. H. CARTER, Professor of Mathematics, Centenary College of Louisiana, Jackson, La.

Let $10x_{10}+9$=the number, also, $9x_9+8$; $8x_8+7$; $7x_7+6$; $6x_6+5$; and so on to $2x_2+1$=the number.

$\therefore 9x_9+8=10x_{10}+9. \quad \therefore x_9=x_{10}+\frac{1}{9}(x_{10}+1).$

But x_9 and x_{10} are both integral.

$\therefore (x_{10}+1)/9=m$ an integer. $\quad \therefore x_{10}=9m-1=(90m/10)-1.$

The value of x_9 from the above equation is $(90m/9)--1$.

Similarly for the other values, the expression $x_n=(90m/n)-1$, giving one of the values for each value of n from 10 to 2. But since all these values are to be integral, $90m$ must be a multiple of each of the natural numbers from 2 to 10 inclusive. This requires m to be $4\times 7=28$, or some multiple of 28. If $m=28$, $x_{10}=251$.

$\therefore 10x_{10}+9=2519$=one of the numbers.

Taking m=the multiples of 28, we get other numbers, 5039, 7559, 10,079. Still other numbers can be obtained by taking the higher multiples of 28 for m.

A similar solution gives for second statement, the number 80,313,433,199.

V. Solution by O. W. ANTHONY, M. Sc., Columbian University, 1702 S Street, Washington, D. C.

The problem in question may be generalized thus : Find a number such that if it be divided by a particular number or any number less than this number the remainder will be one less than the divisor.

Let x be the required number. It is evident, if k and $k+l$ be any two numbers less than the first divisor in question, the following conditions must be satisfied :

$$x/k=u_1+(k-1)/k.....(1). \qquad k/(k+l)=u_2+(k+l-1)/(k+l)..... (2),$$

or $x=ku_1+k-1.....(3)$, and $x=(k+l)u_2+k+l-1 .....(4).$

Take the value of x given in (3) and substitute it for u_2 in (4). Then

$x=(k+l)[ku_1+k-1]+k+l-1$.....(5), which may be reduced to the following form: $x=k[(k+l)u_1+k+l-1]+k-1$.....(6).

Thus (5) and (6), which are identical, contain both the forms (3) and (4). Thus if we substitute in the manner indicated the result will contain two original forms. Some special forms required by the problem in question are:

$x=2u_1+1$....(1); $x=3u_2+2$....(2); $x=4u_3+3$....(3); $x=5u_4+4$....(4);

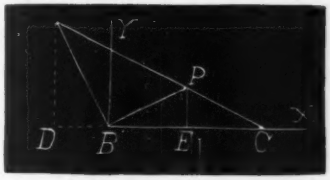
etc., etc. Substitute (1) in (2) in the manner indicated above and we have $x=6u_1+5$. This includes (1) and (2). Substitute this in (3); the result is $x=24u_1+23$. This includes (1), (2), and (3) by the previous demonstration. Continuing this we have as a result $x=|\,ku_1+|\,k-1=|\,k(u_1+1)-1$.....(A). This contains forms (1), (2), (3), (4), etc., and is the general form of number required. The examples cited are special applications of this general form. Thus $x=|\,8(u_1+1)-1$ contains all the numbers required in the first part of the problem, and, letting $u_1=0$, and $k=25$, we have $x=|\,25-1$, the number required in the last part of the problem.

46. Proposed by A. H. HOLMES, Box 963, Brunswick, Maine.

The base BC of the triangle ABC is $2c$, the sum of the two sides, AB and BC, is $2a$. BP is always perpendicular to AB and cuts AC in P. What is the locus of the point P?

I. Solution by GEORGE LILLEY, Ph. D., LL. D., 394 Hall Street, Portland Ore.

Take BC for the axis of x; let P be (x, y); draw AD at right angles to BC, produced; and PE at right angles to BC.



Area ABP+area PBC=area ABC, or

$(a-c)|\sqrt{x^2+y^2}+cy=c\times AD$..... .....(1).

Triangles ABD and BPE are similar.

Hence, $AD=[2y(a-c)]/|\sqrt{x^2+y^2}$.....(2).

From (1) and (2), $(a-c)(x^2+y^2)+cy|\sqrt{x^2+y^2}=2cy(a-c)$.

$\therefore c^2y^2(x^2+y^2)=(a-c)^2(2cy-x^2-y^2)$, for the required locus.

If $\angle ABC$ be an acute angle, y must be taken negatively. Then, area ABC +area BPC=area ABP, or

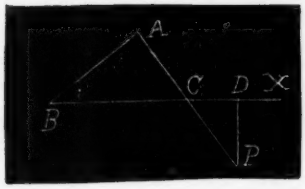
$$c\times AD+c(-y)=(a-c(|\sqrt{x^2+y^2} \quad .....(3),$$

$$\text{and } AD=[2x(a-c)]/|\sqrt{x^2+y^2} \quad .....(4).$$

From (3) and (4), $c^2y^2(x^2+y^2)=(a-c)^2(2cx-x^2-y^2)^2$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Lebanon, Va.

Let B be the origin, Bx the initial line, $BP=r$, $\angle CBP=\theta$, $BC=2c$, $AB=2(a-c)$.



Then $\cos B=\sin\theta$.

$$AC=\sqrt{8c^2-8ac+4a^2-8c(a-c)\sin\theta}.$$

$$CP=\sqrt{4c^2-4rc\cos\theta+r^2}.$$

$$r^2+4(a-c)^2=AP^2=(AC+CP)^2.$$

Substituting and reducing we get for the locus

$$4c^2(r\cos\theta+2a\sin\theta-2c-2c\sin\theta)^2$$

$$=(4c^2-4rc\cos\theta+r^2(\{8c^2-8ac+4a^2-8c(a-c)\sin\theta\}.$$

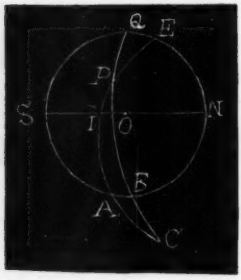
Also solved by A. H. BELL.

47. Proposed by S. HART WRIGHT, A. M., Ph. D., Penn Yan, New York.

In longitude 75 degrees west of Greenwich, latitude 43 degrees, 30 minutes north on January 1, 1895, at 3 o'clock A. M., local time. What points of the ecliptic were then rising, setting and on the meridian? Any other necessary data may be taken from an ephemeris.

Solution by the PROPOSER.

January 1, 1895, 3 A. M., in local mean time, at the station, is December 31, 1894, 15th hour astronomical time. And 15h.+5h., the longitude=20h., mean solar time=20h. 3m. 17.1295s. of sidereal time. To this add from ephemeris sidereal time of mean noon at Greenwich, 18h. 39m, 36.83s., and we have 14h. 42m. 52.9595s.=h, the sidereal time at station. The vernal equinox is then h hours *west* of the meridian of station, or 24h.$-h$ *east* of it, and therefore $24-(h+6)$=3h. 17m. 7.0405s.=49° 16′ 45.6″=a, east of, and below the east point of the horizon of the station.



Let $NQSBN$ be the horizon of station, $EIAC$ a portion of the ecliptic, $QOBC$ a portion of the equator, C the place of the vernal equinox, A the rising point of the ecliptic, I the point then on the meridian, and E the setting point, P the autumnal equinox, and B the point east of the horizon.

Then $BC=a$, the angle BCA, IPO=obliquity of ecliptic=23° 27′ 19″ per ephemeris for the date. The angle ABC=90°+the latitude of station=133° 30′. In the spherical triangle ABC, we have, therefore, the angles B and C given and the side a=49° 16′ 45.6″ to find the side $AC=b$. By spherical trigonometry, b=73° 45′ 15″. In the right spherical triangle IPO, right angled at O, we have $h-12$ hours=2h. 42m. 52.9595s.=PO=40° 43′ 14.4″. By spherical trigonometry, PI=43° 10′ 36″. Hence the rising point is $360°-b$=286° 14′ 45″ of the

ecliptic, and as great circles intersect in opposite points, E will be 180° less than A, or 106° 14′ 45″, and $180° + PI = 223°\ 10'\ 36''$, the longitude of the point passing the meridian.

The senseless divinations of Astrology, are almost entirely based upon finding the three points of the ecliptic required in this problem, for the moment of birth, at a given place.

Also solved by *EDMUND FISH*, Hillsboro, Ill.

48. Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Penn.

In case of *mischance*, with what force would the cow, weighing $w = 700$ pounds, jumping over the moon, have struck Her Lunar Majesty in the face?

Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, Russell College, Lebanon, Va.

Let m=mass of cow on moon, $g' = \frac{1}{6}g$=gravity on moon, r=2163 miles=radius of moon, a=238840 miles=distance from earth to moon, A=momentum $= mv$, E=kinetic energy$= \frac{1}{2}mv^2$.

$$\text{Then } v^2 = 2g'r\left(\frac{a-r}{a}\right), \quad m = \frac{700}{6g'}.$$

$$\therefore\ A = 700\sqrt{\frac{2r}{ag'}(a-r)} = 700\sqrt{\frac{3r}{ag}(a-r)},$$

$$= 700\sqrt{\frac{6489 \times 5280 \times 236677}{238840 \times 32.2}} = 239595.79 \text{ foot-pounds.}$$

$E = (350r/3a)(a-r) = 1320341350.762$ foot-pounds.

The value of A is the force required.

PROBLEMS FOR SOLUTION.

ARITHMETIC.

83. Proposed by the late REV. G. W. BATES, A. M., Pastor of M. E. Church, Dresden City, Ohio.

A has three notes; the first and second, $1000 each, and the third $457; all dated April 1, 1884. The first is due April 1, 1888, second, April 1, 1889, and the third, April 1, 1890, and each bearing interest at 6%. What must B pay for the three notes September 21, 1886 that the investment will bring him 8% compound interest?

[NOTE—The above problem was the result of an actual business transaction.]

84. Proposed by SYLVESTER ROBBINS, North Branch Depot, N. J.

Show how to find sides, integral, fractional, and irrational for twenty-four triangles, each one containing 330 square yards.

85. Proposed by E. W. MORRELL, A. M., Professor of Mathematics, Montpelier Seminary, Montpelier, Vt.

In turning a one-horse chaise within a ring of a certain diameter, it was observed that the outer wheel made two turns, while the inner wheel made but one. The wheels were each 4 feet high; and supposing them fixed at the distance of 5 feet on the axletree, what was the circumference of the track described by the outer wheel? From *Greenleaf's National Arithmetic.*

86. Proposed by EDGAR H. JOHNSON, Professor of Mathematics, Emroy College, Oxford, Ga.

$\frac{1}{7} = .\dot{1}42857\dot{}$; $\frac{1}{11} = .\dot{0}\dot{9}$; $\frac{1}{13} = .\dot{0}76923\dot{}$; $\frac{1}{17} = .0588235294117647\dot{}$.

Observe that if the numbers forming the first half of the repetend be added respectively to the numbers forming the second half of the repetend, the sum is in every case 9. What is the general law of which these are special cases?

GEOMETRY.

80. Proposed by J. C. GREGG, Superintendent of Schools, Brazil, Ind.

One circle touches another internally, and a third circle whose radius is a mean proportional between their radii passes through the point of contact. Prove that the other intersections of the third circle with the first two are in a line parallel to the common tangent of the first two. [From *Phillips and Fisher's Geometry.*]

81. Proposed by CHAS. C. CROSS, Laytonsville, Md.

A circle is drawn bisecting the lines joining the points of contact of the inscribed circles with the sides produced. Another circle is drawn passing through the centers of the circles drawn tangent externally to the in-circle and internally to the sides of the triangle. Prove that the centers of these two circles, the incenter and the circumcenter are collinear.

82. Proposed by WILLIAM SYMMONDS, A. M., Professor of Mathematics and Atronomy, Pacific College, Santa Rosa, Cal.

If the extremities of the base of a triangle be joined by straight lines to the exterior angles of squares constructed upon its two sides, the superior pair of lines thus drawn intersect at right angles; the inferior pair intersect at a point in a line drawn from the vertical angle perpendicular to the base.

MECHANICS.

58. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University, Miss.

An endless uniform chain is hung over two small smooth pegs in the same horizontal line. Show that, when it is in a position of equilibrium, the ratio of the distance between the vertices of the two catenaries to half the length of the chain is the tangent of half the angle of inclination of the portions near the pegs. [From *Ruth's Analytical Statics. Mathematical Trifos, 1855.*]

59. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

Find the radius of sphere of given specific gravity which will rest just immersed in a fluid whose density varies as its depth.

60. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

What must be the ratio of the two legs of a uniform and heavy right triangle suspended from the center of the inscribed circle, if this triangle will rest with the shorter leg in a horizontal position?

AVARAGE AND PROBABILITY.

57. Proposed by G. B. M. ZERR, A. M., Ph. D., Presfdent and Professor of Mathematics, Russell College, Lebanon, Va.

A chord is drawd through two points taken at random in the surface of a circle. If a second chord be drawn through two other points taken at random in the surface, find the chance that the quadrilateral formed by joining the extremeties of the two chords will contain the center of the circle.

58. Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.

From a point on the surface of a circle two lines are drawn to the circumference. Required the average area that may be cut from the circle in this way if the lines are supposed to be drawn at equal angular intervals.

Query I. How does this differ from problem 32?

Query II. Is *sector* the proper word to use for the surface thus cut off?

Query III. It is absolutely correct to use the word *random* in average problems?

NOTES.

THE INTERNATIONAL MATHEMATICAL CONGRESS.

The meeting at Zurich, August 9th–11th, of the International Congress of Mathematicians was in every way a success. More than two hundred members took part. America sent seven representatives, including, however, three Cambridge graduates, now transplanted to Pennsylvania, Professors Harkness, Morley and Charlotte Scott. The greatest mathematician in the world, Sophus Lie, was not expected; and the greatest French mathematician, Poincaré, though down for a speech, did not come; but the actual program was particularly rich and interesting.

It is very noteworthy that the Congress was divided into five sections: (1) Arithmetic and Algebra; (2) Analysis, and Theory of Functions; (3) Geometry; (4) Mechanics and Mathematical Physics; (5) History and Bibliography.

The program of the first section contained the only title in English: "On Pasigraphy, its present state and the pasigraphic movement in Italy," by Ernst Schroeder, of Karlsruhe, author of "Algebra der Logik."

The second section contained a title from Z. de Galdeano, whose heroic efforts gave Spain a Journal of Mathematics, now unfortunately dead in the decadence of that beautiful, priest-ridden land.

The program of the third section, the only one consecrated wholly to a single title, Geometry, contained two titles on the non-Euclidean geometry.

Burali: Les postulats pour la geométrie d'Euclide et de Lobatschewsky.

Andrade: 'La statique non euclidienne et diverses formes mécaniques du postulatum d'Euclide.

In Section IV. Stodola treated an important subject, "Die Beziehungen der Technik zur Mathematik."

In the fifth section Eneström gave an important discussion of bibliography, a point where the Congress can and will render aid of fundamental importance.

In the first general assembly Rudio spoke on the aim and organization of international mathematical congresses.

It was determined that the next Congress should take place at Paris in 1900, under the auspices of the Société Mathématique de France.

As aims were specified : (1) to promote personal relations between mathematicians of different lands ; (2) to give, in reports or conferences, an aperçu of the actual state of the divers branches of mathematics, and to treat questions of recognized importance ; (3) to deliberate on the problems and organization of future congresses ; (4) to treat questions of bibliography, of terminology, etc., on subjects where an *entente international* appears necessary.

Rudio mentioned the yearly issue of an address-book of all the mathematicians of the world with indication of their specialties ; also of a biographic dictionary of living mathematicians with portraits ; also of a literary journal for mathematics.

At the second general assembly Peano gave a conference : "Logica matematica"; and Felix Klein a conference on teaching higher mathematics.

Three important resolutions were introduced by Vasiliev, of Kazan ; Laisant, of Paris, and G. Cantor, of Halle, constituting : (1) a commission for preparation of general reports ; (2) a standing bibliographic and terminology commission ; (3) a commission to give the congress a permanent character by archives, libraries, stations for correspondence, editing or publishing noteworthy works, etc.

Surely this Congress has proven that it came only in the fullness of time, and that the world moves !

GEORGE BRUCE HALSTED.

Austin, Texas.

EDITORIALS.

Dr. O. E. Lovett has been called to Princeton University as Assistant Professor of Mathematics.

Dr. George Lilley, LL. D., has been elected to the Chair of Mathematics in the State University of Oregon.

A portrait of a group of five of our contributors will appear soon. We were unable to complete the arrangements for this number.

Dr. L. E. Dickson, who spent last year at the Universities of Göttingen and Paris, has been elected Assistant Professor of Mathematics in the University of California.

Miss Mary F. Winston, Ph. D., has been elected Professor of Mathematics at the Kansas State Agriculturist College, Manhattan, Kansas.

Prof. E. D. Roe, Jr., Assistant Professor of Mathematics in Oberlin College, is taking a two years course in mathematics, in Göttingen, Germany.

Professor D. A. Lehman, the past year Professor of Mathematics in the College of the Pacific, has been called to the Chair of Mathematics in the Balwin University, Berea, Ohio.

The biography of Professor J. J. Sylvester which appeared in the June-July number of the MONTHLY has been translated in Russian and published by Professor Vasiliev, the great Russian Mathematician.

We regret to record the death of one of our valued contributors, De Volson Wood, Professor of Mechanical Engineering at the Steven Institute of Technology, Hoboken, N. J., on June 27, at the age of sixty-five years. We take pleasure in giving our readers a short account of his life in this issue.

We are pleased to state that we have in our hands Dr. Lovett's first article on Sophus Lie's Transformation Groups, which will surely appear in our next issue. It is Dr. Lovett's purpose to make the series of articles very elementary at first and thus bring this most important subject within the comprehension of the most of our readers. These articles alone will be worth many times the price of subscription to the MONTHLY.

BOOKS AND PERIODICALS.

The Non-Regular Transitive Substitution Groups whose Order is the Product of Three Unequal Prime Numbers. Reprint of a paper in *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich.* By Dr. G. A. Miller, Paris, France. 6 pages. B. F. F.

A History of the United States. By Allen C. Thomas, A. M., Professor of History in Haverford College, Penn. 8vo. cloth and leather back. 418 and lxxiv pages. Boston : D. C. Heath & Co.

This is the best school history of the United States that has yet been published. B. F. F.

The Tutorial Statics. By William Briggs and G. H. Bryan. 260 pages. Price, $1.00. London : W. B. Clive. New York : Hinds and Noble.

The plan of this work is good and the execution satisfactory. With the exception of some looseness of statement in certain paragraphs, the work is well written and should prove serviceable for class use. There are many valuable hints, explanations and alternative proofs, and a large selection of examples, throughout the text. An excellent summary of results follows each chapter. J. M. C.

Grammar School Arithmetic by Grades. Edited by Eliakim Hastings Moore, Ph. D., Head Professor of Mathematics, The University of Chicago. 8vo. cloth. 352 pages. Price, 60 cents. Chicago : American Book Co.

Some of the prominent features of this work are, the accurate definitions of terms according to modern usage, the use throughout of the inductive or laboratory method, the numerous well selected problems, and the entire absence of rules. The treatment of arithmetic as given in this book is a definite departure from the old ruts, and we believe that the timely appearance of this work will go far towards correcting many of the vicious and unwholesome methods pursued in many schools. B. F. F.

Elementary Text-Book of Physics. By Prof. Wm. A. Anthony, formerly of Cornell University, and Prof. Cyrus F. Brackett, of Princeton University. Revised by Prof. William Frances Magie, of Princeton. Eighth edition, revised. 8vo. cloth. 512 pages. Price, $3.00. New York and London : John Wiley & Sons.

This work deserves especial praise for the direct and logical manner in which it discusses the fundamental principles of Physics. The pictorial representations of apparatus are purposely omitted as are also the illustrations of the fundamental principles by detailed description of special methods of experimentation and of devices necessary for their applications in the arts, and thus space is saved for the discussion of important principles.

The work is admirably adapted to those schools and colleges having a large collection of apparatus, but for those that have but few pieces of apparatus, the absence of pictorial representations in a text book would in many cases leave the student without any ideas at all as to their construction. B. F. F.

Theory of Physics. By Joseph S. Ames, Ph. D., Associate Professor of Physics and Sub-Director of the Physical Laboratory in Johns Hopkins University. Crown 8vo. cloth. 514 pages. Price, $1.60 ; by mail, $1.75. New York : Harper and Brothers.

"To present successfully the subject of Physics to a class of students, three things seem to me as necessary: a text-book, a course of experimental demonstrations and lectures, accompanied by recitations, and a series of laboratory experiments, mainly quantitative, to be performed by the students themselves under the direction of instructors. I place "text-book" first, because for many reasons I believe it to be the most important of the three. None but advanced students can be trusted to take accurate and sufficient notes of lectures; and a text-book which states the theory of the subject in a clear and logical manner so that recitations can be held on it, seems to me to be absolutely essential." *Preface.*

This work which has just recently been issued discusses in a most satisfactory manner, the latest discoveries made in Physics. The doctrines of energy are stated with the utmost clearness and are made the framework for a consecutive treatment of Physics as a whole. The strong points in favor of this book are too numerous to mention in the limited space at our disposal. B. F. F.

The New Arithmetic. Part Part One for Teachers. By William W. Speer, Assistant Superintendent of Schools, Chicago. 154 pages. Boston and London: Ginn & Co. 1897.

This book is one of a series now in press. Some rather radical departures are proposed. The author thinks that the study of Arithmetic should be advanced from the science of *number* to that of the *definite relations of quantity.* The book gets the idea of *technical measurement* in early. Simple ratios are made the key to the solution of all problems.

The quotations in support of the theory of the book it seems to us are carried to excess. We doubt if the representation of *cents* by *lines*, p. 118, leads to clear ideas of relative values, and the "guessing" exercise on page 42 seems rather ludicrous. Notwithstanding minor objections the book is undoubtedly one of many excellencies, and the appearance of the other books of the series will be awaited with more than usual interest. J. M. C.

Mathematical Questions and Solutions. From the "Educational Times," with an Appendix. Edited by W. J. C. Miller, B. A. Vol. LXVI. 128 pages. Francis Hodgson, 89 Farringdon Street, E. C., London.

This valuable reprint contains solutions of 145 interesting problems. The price is 5s. 3d., postpaid. J. M. C.

Descriptive Geometry. Straight Line and Curves. By William J. Meyers, Professor of Mathematics in the State Agricultural College of Colorado, Fort Collins, Colo. Pages, 66 and several pages of excellent Plates. Printed by the Author.

The author has aimed to strike a mean between an abstract and difficult treatment and a diffuse and easy one. The method is based on the authors experience in his class room. The book is well supplied with suitable exercises, and deserves careful examination on the part of teachers who have occasion to use an elementary text on this subject. J. M. C.

Introduction to Infinite Series. By William F. Osgood, Ph. D., Assistant Professor of Mathematics in Harvard University. 71 pages. Cambridge: Published by Harvard University. 1897.

This little book deals with an important topic. The presentation aims to acquaint the student with the nature and use of these series and to introduce him to the theory in such a way that at each step he sees precisely the question at issue. As aids to this end the work gives a variety of illustrations of applications of these series to computations in pure and applied mathematics, a full and careful exposition of the meaning and scope of the more difficult theorems, and the use of diagrams and graphical illustrations in the proofs. We have read these chapters with much interest and heartily commend the book to our readers as a valuable supplement to the treatment given in the usual text-books on the Differential and Integral Calculus. J. M. C.

Intermediate Algebra. University Tutorial Series. By William Briggs, M. A., F. C. S., F. R. A. S., and G. H. Bryan, Sc. D., F. R. S. 375 pages. Price, $1.00. London: W. B. Clive. New York Depot: Hinds and Noble.

This is a work of more than ordinary merit. It is based on the treatise of Radhakrishnan, with such alterations and additions as were necessary to render it suitable to the wants of English and American students. The simple properties of Inequalities are treated at an early stage, the important properties of Zero and Infinity are adequately presented, and the theory of Quadratic expressions and Maxima and Minima are fully discussed. The chapters on Logarithms, Interest and Annuties are excellent in every detail. J. M. C.

Elementary and Constructional Geometry. By Edgar H. Nichols, A. B., of the Brown and Nichols School, Cambridge, Mass. Pages 138. New York: Longmans, Green & Co.

This book is very carefully written and is admirably adapted for the place it is designed to fill. The author uses the words *symparallel* and *antiparallel* for parallel lines

that have the same and the opposite directions, respectively. A proper use of the blank pages at the end of the book for a summary of facts, definitions, and principles will add greatly to the usefulness of the book. J. M. C.

The Science of Mechanics. A Critical and Historical Exposition of Its Principles. By Dr. Ernst Mach, Professor of Physics in the University of Prague. Translated from the Second German Edition by Thomas J. McCormack. With two hundred and fifty cuts and illustrations. Half morocco, gilt top, marginal analysis, exhaustive index. Price, $2.50. Chicago : The Open Court Publishing Co.

This is one of the most readable works on Mechanics that has yet come to our notice. The rigorous and rigid mathematical reasoning is interspersed by many interesting historical facts concerning the application and development of the principles under consideration, as well as giving some pleasing accounts of the first discoveries of these principles. The work is in every way worthy the highest patronage, and no difference what text-book on Mechanics may be adopted for class use, Dr. Mach's book ought to be in use in every class to supplement the work of the regular course. The book is beautifully printed and handsomely bound. B. F. F.

Elementary Mathematical Astronomy. With Examples and Examination Papers. By C. W. C. Barlow, M. A., B. Sc., Gold Medalist in Mathematics at London M. A.; Sixth Wrangler, and First Class First Division Part II. Mathematical Tripos, Cambridge, and G. H. Bryan, M. A., Sc. D., F. R. S., Smith's Prizeman, Fellow of St. Peter's College, Cambridge ; Joint Author's of "Coördinate Geometry." 16mo. cloth. 442 pages. Price, $1.50. London : W. B. Clive, University Correspondence College Press ; and New York : Hinds and Noble.

Nothing but words of praise can be said of this work. A somewhat careful examination leads us to pronounce it the best in the particular field it is designed to cover. The book gives a most excellent description of the methods by which the structure of Scientific Astronomy has been built up with a very small amount of mathematical knowledge. The book should be the delight of every student of Astronomy. The arrangement is good, the diagrams clear and accurate, and the whole treatment excellent. B. F. F.

The Open Court. A Monthly Magazine devoted to the Science of Religion, the Religion of Science, and the Extention of the Religious Parliament Idea. Edited by Dr. Paul Carus ; T. J. McCormack, Assistant Editor ; E. C. Hegeler. and Mary Carus, Associate Editors. Price, $1.00 per year in advance. The Open Court Publishing Co., Chicago, Ill.

Among the articles in the August number are the following: The Religion of Islam, by Hyacinthe Loyson; History of the People of Israel, from the Beginning of the Destruction of Jerusalem, by Dr. C. H. Corniell, Professor of Old Testament History in the University of Kōnigsberg; and the Evolution of Evolution, by Dr. Moncure D. Conway.
B. F. F.

The Cosmopolitan. An International Illustrated Monthly Magazine. Edited by John Brisben Walker. Price, $1.00 per year in advance. Single number, 10 cents. Irvington-on-the-Hudson.

The Mathematical Gazette. Edited by F. S. Macauley, St. Paul's School, West Kensington, London. Issued three times a year, viz : in February, June, and October. Price, one shilling, net.

The June number contains an article on Spherical Geometry: I. Orthogonal Projection, by Prof. Alfred Lodge, M. A.; II. Stereographic Projection, by P. J. Heawood, M. A. Also Notes, Mathematical Notes, Examination Questions and Problems, Solutions, and Reviews and Notices. In "Notes" is an extended notice of Dr. Halsted's article on the "Non-Euclidean Geometry" which appeared in the March number of the MONTHLY.

B. F. F.

The Monist. A Quarterly Magazine devoted to the Philosophy of Science. Edited by Dr. Paul Carus ; T. J. McCormack, Assistant Editor ; E. E. Hegeler, and Mary Carus, Associate Editors. Price, $2.00 per year in advance. Single number, 50 cents. The Open Court Publishing Co., Chicago, Ill.

The following articles appeared in the January, 1897, number: The Logic of Relatives, by Chas. S. Peirce; Man as a Member of Society, Introduction, by Dr. P. Topinard; The Philosophy of Budhism, by Dr. Paul Carus; Panlogism, by E. Douglas Fawcett; The International Scientific Catalogue, and the Decimal System of Classification, by Thomas J. McCormack; and Literary Correspondence—France, by Lucien Arréat. B. F. F.

The American Monthly Review of Reviews. An International Illustrated Monthly Magazine. Edited by Dr. Albert Shaw. Price, $2.50 per year in advance. Single Number, 25 cents. The American Monthly Review of Reviews Co., 13 Astor Place, New York City.

We are pleased to note that since our last issue this valuable magazine has changed its name to *The American Monthly Review of Reviews*, a more significant title than its former one.

The September number has a good deal to say about the Andrews incident and Brown University—not so much, as the editor remarks, on account of the personal interests involved in the case, as because of the far-reaching principles affecting academic life and liberty which have become matters at issue. A fair-minded and judicious estimate of President Andrews' services to Brown is given by a writer fully conversant with the facts, and the protest of the faculty is printed in full. The editorial comments on the awkwardness and needlessness of the situation are piquant and to the point.

Among the contributed articles in the September number are sketches of the three members of the new Nicaragua Canal Commission—Admiral Walker, Capt. O. M. Carter, Corps of Engineers, U. S. A., and Prof. Lewis M. Haupt. These sketches are illustrated with portraits, and serve to convey an idea of the peculiar qualifications possessed by these gentlemen for the task to which they have been appointed by President McKinley.

B. F. F.

The Arena. An Illustrated Monthly Magazine. Edited by John Clark Ridpath, LL. D. Price, $2.50 per year in advance. Single number, 25 cents. Boston : The Arena Co.

Every true American citizen should read Dr. John Clark Ridpath's splendid paper, "The Cry of the Poor," and his "Open Letter" to President E. B. Andrews, which appear in the September number of the *Arena*. In them the Doctor has drawn a picture that appeals to every man and woman in our land who has God-given rights and privileges which, owing to the intervention of plutocratic influences, they are not allowed to enjoy.

"Why," asks the Doctor, "should the voice of the poor ever be heard rising like a wail from plantation, hamlet, and cityful ? Why should there be seen standing at the

door of the homes of the American people the gaunt spectre—Want ?" "And why," he again asks, "should we allow the voice of our teachers to be smothered by plutocratic powers ?" There may be those who sanction the conduct of Brown University in expelling Professor Andrews, but it is very evident that the editor of the *Arena* and the author of "The Bond and the Dollar" and "The True Inwardness of Wall Street" does not.

Among the other papers are "The Concentration of Wealth, its Cause and Results: Part I," by Herman E. Taubeneck; "The Multiple Standard for Money," by Eltweed Pomeroy; "The Future of the Democratic Party: A Reply," by David Overmyer; "The Author of 'The Messiah'," by B. O. Fowler; "Anticipating the Unearned Increment," by I. W. Hart; "Studies in Ultimate Society:" I. "A New Interpretation of Life," by Laurence Gronlund; II. "Individualism vs Altruism," by K. T. Takahashi; "General Weyler's Campaign," by Crittenden Marriott; the "Plaza of the Poets," "Book Reviews," and "The Editor's Evening," make up this bright and instructive number.

CORRECTIONS AND REVISIONS OF THE ARTICLE "ON THE CIRCULAR POINTS AT INFINITY,"

MAY MONTHLY, PP. 132—145.

(P.=page ; l. κ=κth line from above ; lb. κ=κth line from below.)

P. 132, l. 1 of the article, read Coördinate for Coordinate ; l. 2, Cartesian for Cartesion. P. 133, (4) and (4)′ for (A) and $(A)'$; l. 19—21, finish parenthesis ; l. 23, = for −. P. 134, l. 2, vanishes for vanisnes ; interchange lines 14 and 15. P. 135, l. 4, bring "all true" down to l. 6 ; l. 12, add "and" after "infinity"; l. 19 and 23, coördinates for coöordinates ; l. 25, coördinates for coödinates. P. 136, l. 4, add exponent 2 to numerator ; l. 6, ρ^2 here taken equal to 1, might have been retained in the numerator. If retained, (21) p. 140 would contain ρ^4 instead of ρ^2, but this would have no effect on the final result (22). Whether ρ^2 is retained or not, (14) would have to be made homogeneous in all the coördinates involved, as well as (21), for practical uses, since this is required of all such equations. (14) can be made homogeneous by the use of the solution of (4). l. 9, $+\sin\alpha_1\sin\alpha_2$ for $-\sin\alpha_1\sin\alpha_2$; $-\kappa_1\kappa_2\cos C$ for $\kappa_1\kappa_2\cos C$; lb. 6, c for C. P. 137, l. 16, r for γ. P. 138, lb. 4, $x_2'^2$ for $x_1'^2$; lb. 5, r for γ ; lb. 8, $\cos C$ for $\cos B$; lb. 9, $x_1'^2$ for x_1^2. P. 139, l. 5, κ_3^2 for x_3^2 and for κ_3 ; l. 12, $x_3'x_1$ for x_3x_1. P. 140, lb. 2, $(x'\xi\xi')^2$ for $(x'\xi\xi)^2$. P. 141, l. 14, x' for x ; l. 17, $\kappa_2^2u_2^2+\kappa_3^2u_3^2$ for $\kappa_2u_2^2+\kappa_3u_3^2$; lb. 1, $x_2'^2$ for x_2' ; in foot note, "Nicht-Euklidische Geometrie" for "Nicht-Euclidsche Geometry." P. 142, l. 9, $-iA$ for $-iB$; l. 11, *two lines* for *the lines* ; l. 13 and 14, x and y might be interchanged, though this is not necessary ; the other angle between the two lines would be given ; l. 15, The double ratio of these is : Taking them in the order named, using etc.; l. 18, s for 5 ; l. 19, $+s\lambda'$ for $+s\lambda$. P. 144, l. 11, $\tan\psi$ forψ ; slopes for tangents would be better ; l. 12, it is necessary and sufficient that the purely imaginary part of x should become indefinitely great ; l. 18, the German word "quadrupel" is here appropriated ; l. 23, ±1 for $\pm l$; l. 27, is for in ; in foot note, * for †. P. 145, l. 4, $\Sigma xx.\Sigma x'x'$ for$\Sigma xx'.\Sigma x'x$ in numerator and denominator ; $(\Sigma xx')^2$ for $\Sigma xx'$ under radical in denominator ; l. 5, two points for the points ; l. 7, Σxx for $\Sigma xx'$.

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